6. A rectangle is inscribed in an equilateral triangle, as shown in the diagram below, with a perimeter of 30 cm. Express the area of the rectangle as a function of x.
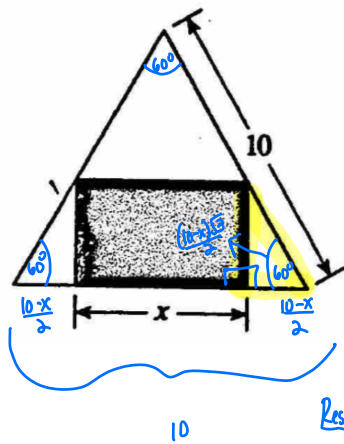
$A = bh$

$A(x) = x \cdot h$ ← need h in terms of x

$A(x) = x \cdot \dfrac{(10-x)\sqrt{3}}{2}$, $(0,10)$
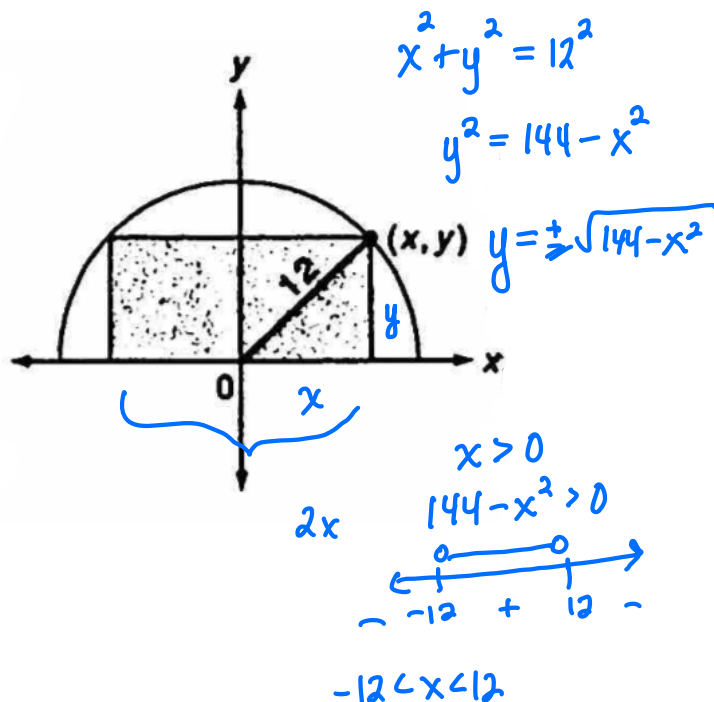
Restrictions
$x > 0$
$x < 10$

Homework 11-14

Name: _____
PCH: Modeling with Functions Practice Packet 4

Date: _____
Ms. Loughran

1. A rectangle is inscribed in a semicircle of radius 12 as shown. Express the area of the rectangle as a function of x.
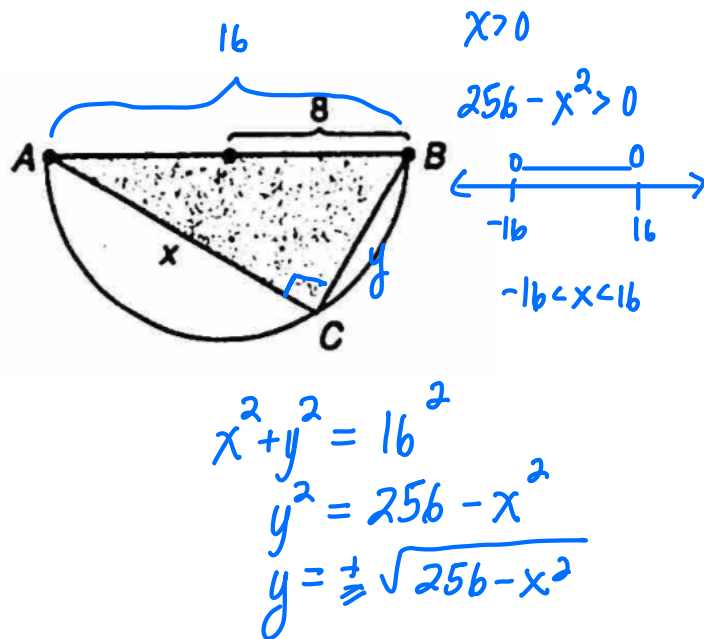
$$A = lw$$
$$A(x) = 2x \cdot \sqrt{144 - x^2}, \quad 0 < x < 12$$

$$x^2 + y^2 = 12^2$$
$$y^2 = 144 - x^2$$
$$(x,y) \quad y = \pm\sqrt{144 - x^2}$$

$$x > 0$$
$$2x \qquad 144 - x^2 > 0$$

$$-12 < x < 12$$

2. Triangle ABC is inscribed in a semicircle of radius 8 so that one of its sides coincides with a diameter. Express the area of the triangle as a function of $x = AC$.

$$A = \tfrac{1}{2}bh$$
$$A(x) = \tfrac{1}{2}x \cdot \sqrt{256 - x^2}, \quad 0 < x < 16$$

$$16$$
$$8$$

$$x > 0$$
$$256 - x^2 > 0$$

$$-16 < x < 16$$

$$x^2 + y^2 = 16^2$$
$$y^2 = 256 - x^2$$
$$y = \pm\sqrt{256 - x^2}$$

3. *ABCD* is an isosceles trapezoid in which sides *AB* and *DC* are parallel. Express the area of the trapezoid as a function of altitude *h*.
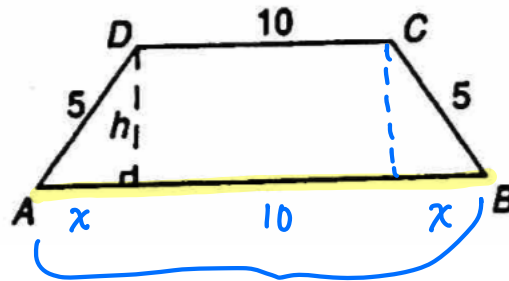
$h > 0$

$25 - h^2 > 0$

$$A = \tfrac{1}{2}h(b_1 + b_2)$$

$$A(h) = \tfrac{1}{2}h\left(10 + 10 + 2\sqrt{25 - h^2}\right)$$

$$A(h) = \tfrac{1}{2}h\left(20 + 2\sqrt{25 - h^2}\right)$$

$$A(h) = 10h + h\sqrt{25 - h^2}$$
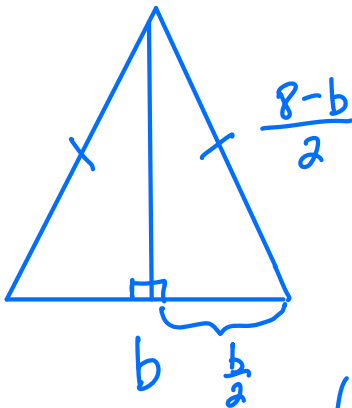
$$0 < h < 5$$

$$x^2 + h^2 = 5^2$$

$$x^2 = 25 - h^2$$

$$x = \pm\sqrt{25 - h^2}$$

$$10 + 2x$$

$$10 + 2\sqrt{25 - h^2}$$

$-5 < x < 5$

4. An isosceles triangle has a perimeter of 8cm. Express the area *A* of the triangle as a function of the length *b* of the base of the triangle.

$$A = \tfrac{1}{2}bh$$

$$A(b) = \tfrac{1}{2}b \cdot \left(2\sqrt{4 - b}\right), \quad 0 < b < 4$$

$$A(b) = b\sqrt{4 - b}$$

$$\lg = \frac{8-b}{2}$$

$$\frac{8-b}{2}$$

$$b > 0$$

$$4 - b > 0$$

$$-b > -4$$

$$b < 4$$

$$\left(\tfrac{b}{2}\right)^2 + h^2 = \left(\tfrac{8-b}{2}\right)^2$$

$$\frac{b^2}{4} + h^2 = \frac{(8-b)^2}{4}$$

$$h^2 = \frac{(8-b)^2}{4} - \frac{b^2}{4}$$

$$h^2 = \frac{(8-b)^2 - b^2}{4}$$

$$h = \pm\sqrt{\frac{(8-b)^2 - b^2}{4}}$$

$$h = \sqrt{\frac{64 - 16b + b^2 - b^2}{4}}$$

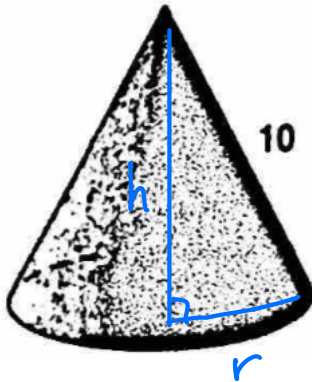$$h = \sqrt{\frac{16(4-b)}{4}}$$

$$h = \sqrt{4(4-b)} = 2\sqrt{4-b}$$

5. The figure shows a right circular cone in which r is the radius of the base, and the slant height is 10. Express the volume of the cone as a function of r.

$r > 0$

$100 - r^2 > 0$

$-10 < r < 10$

10

h

r

$V = \frac{1}{3}\pi r^2 h$

$V(r) = \frac{1}{3}\pi r^2 \cdot \sqrt{100 - r^2} \quad , \quad 0 < r < 10$

$r^2 + h^2 = 10^2$

$h^2 = 100 - r^2$

$h = \pm\sqrt{100 - r^2}$