Name: ____ Date: __________

PCH: Modeling with Functions Ms. Loughran

$$S_{cylinder} = 2\pi r^2 + 2\pi r h$$

Do Now:

1. A container in the shape of a right circular cylinder with no top has a surface area of 42π ft^2. Write a formula for the volume of the cylinder in terms of its radius, *r*.

$S = \pi r^2 + 2\pi r h$

$42\cancel{\pi} = \cancel{\pi} r^2 + 2\cancel{\pi} r h$

$42 = r^2 + 2rh$

$\dfrac{42 - r^2}{2r} = \dfrac{2rh}{2r}$

$\dfrac{42 - r^2}{2r} = h$

$V = \pi r^2 h \leftarrow$ need h in terms of r

$V(r) = \pi r^2$

$V(r) = \pi r^{\cancel{2}} \cdot \dfrac{(42 - r^2)}{2\cancel{r}}$

$V(r) = \dfrac{42\pi r - \pi r^3}{2}$,

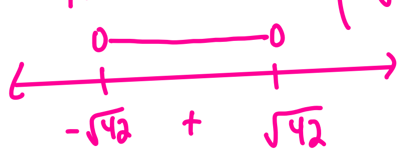
$0 < r < \sqrt{42}$

restrictions

$r > 0$

$\dfrac{42 - r^2}{2r} > 0$

$42 - r^2 > 0$ $(-\sqrt{42}, \sqrt{42})$



Name: ___________________________________ Date: ________

PCH: Polynomials Practice

1. If $f(x)=6x^3-5x^2-17x+6$, find $f\left(\frac{1}{2}\right)$. plug in $\frac{1}{2}$ for x or

$$\begin{array}{r|rrrr} \frac{1}{2} & 6 & -5 & -17 & 6 \\ & & 3 & -1 & -9 \\ \hline & 6 & -2 & -18 & -3 \end{array}$$ remainder

2. If $f(x)=2x^3+5x^2+5px+6$ and $f(2)=12$, find p.

$2(2)^3+5(2)^2+5p(2)+6=12$

$16+20+10p+6=12$

$42+10p=12$

$+10p=-30$

$p=-3$

$$\begin{array}{r|rrrr} 2 & 2 & 5 & 5p & 6 \\ & & 4 & 18 & 10p+36 \\ \hline & 2 & 9 & 5p+18 & 12 \end{array}$$

$6+10p+36=12$

$p=-3$

3. Find the quotient and remainder when $3x^3+x^2-6x+3$ is divided by $3x+1$.

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 1 & -6 & 3 \\ & & -1 & 0 & 2 \\ \hline & 3 & 0 & -6 & 5 \end{array}$$ remainder

$\underbrace{3 \quad 0 \quad -6}_{\div 3}$

$x^2-2+\frac{5}{3x+1}$

4. If f is a polynomial where $f(3)=0$ and $f(-1)=0$, what are two linear factors of f?

$(x-3)(x+1)$

5. Find the zeros of:

(a) $f(x) = x(x+2)(3x-4)$

$x = 0, -2, \frac{4}{3}$

(b) $g(x) = 3x^2 - 9x$

$g(x) = 3x(x-3)$

$x = 3, 0$

(c) $h(x) = 3x^2 - 9x + 7$

$0 = 3x^2 - 9x + 7$

$x = \frac{9 \pm \sqrt{(-9)^2 - 4(3)(7)}}{2(3)}$

$x = \frac{9 \pm \sqrt{81-84}}{6} = \frac{9 \pm \sqrt{-3}}{6} = \frac{9 \pm i\sqrt{3}}{6}$

(d) $j(x) = x^2 + 9$

$0 = x^2 + 9$

$-9 = x^2$

$\pm\sqrt{-9} = x$

$x = \pm 3i$

6. If $x+3$ is a factor of $f(x) = x^3 + 4x^2 + x - 6$, find the complete factorization of $f(x)$.

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 1 & -6 \\ & & -3 & -3 & 6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$f(x) = (x+3)(x^2 + x - 2)$

$f(x) = (x+3)(x+2)(x-1)$

7. Given: $g(x)=2x^4-7x^3-6x^2+44x-40$

(a) Find the multiplicity of the zero 2. 2 has a multiplicity of 3

(b) Factor $g(x)$ completely using integral factors. $(x-2)^3(2x+5)=g(x)$

(c) Find the roots of $g(x)=0$. $\left\{2 \text{ (mult. of 3)}, \frac{-5}{2}\right\}$

$$
\begin{array}{r|rrrrr}
2 & 2 & -7 & -6 & 44 & -40 \\
 & & 4 & -6 & -24 & 40 \\
\hline
2 & 2 & -3 & -12 & 20 & 0 \\
 & & 4 & 2 & -20 & \\
\hline
2 & 2 & 1 & -10 & 0 & \\
 & & 4 & 10 & & \\
\hline
 & 2 & 5 & 0 & &
\end{array}
$$

$g(x)=(x-2)^3(2x+5)$

8. One root of $x^3+4x^2-4x-1=0$ is 1. Find the other roots.

$$
\begin{array}{r|rrrr}
1 & 1 & 4 & -4 & -1 \\
 & & 1 & 5 & 1 \\
\hline
 & 1 & 5 & 1 & 0
\end{array}
$$

$(x-1)(x^2+5x+1)=0$

$x=\frac{-5\pm\sqrt{5^2-4(1)(1)}}{2(1)}$

$x=\frac{-5\pm\sqrt{21}}{2}$

$\left\{\frac{-5\pm\sqrt{21}}{2}\right\}$

9. $F(x)$ is a polynomial function with rational coefficients. What is the minimum degree of $F(x)$ if $\sqrt{2},1,1-\sqrt{2}$ and 3 are zeros of $F(x)$? 6 six

$-\sqrt{2}$ $1+\sqrt{2}$

Thm: If a polynomial has rational coefficients, irrational zeros come in conjugate pairs.

Coefficients are not all real

10. True of False: If $2i$ is a root of $x^2 - \overset{\downarrow}{i}x + 2 = 0$, then $-2i$ is also a root.

Thm: If a polynomial has real coefficients then complex numbers come in conjugate pairs.

11. Find a polynomial $P(x)$ in expanded form with integral coefficients if its zeros are: $\left\{-1, \pm i, \frac{3}{4}(\text{multiplicity of } 2)\right\}$.

$$P(x) = (x+1)(x^2+1)(4x-3)^2$$

$$P(x) = (x^3+x^2+x+1)(16x^2-24x+9)$$

$$P(x) = 16x^5+16x^4+16x^3+16x^2-24x^4-24x^3-24x^2-24x+9x^3+9x^2+9x+9$$

$$P(x) = 16x^5-8x^4+x^3+x^2-15x+9$$

12. Find the remainder when $x^{125} - 5x^{77} + 2x^{46} - 3x + 5$ is divided by $x+1$.

$$(-1)^{125} - 5(-1)^{77} + 2(-1)^{46} - 3(-1) + 5$$

$$-1 - 5(-1) + 2(1) + 3 + 5$$

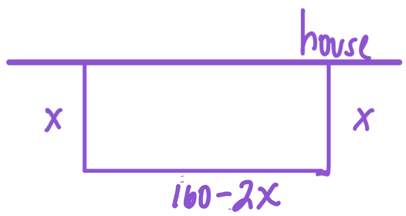
$$-1 + 5 + 2 + 3 + 5$$

$$14$$

Name: ____ Date: _____________

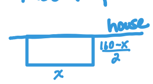
PCH: Modeling with Functions Practice Packet 5

1. Vanessa wants to fence off part of her backyard so her dog Chicken can run around without getting out. She has 160 feet of fencing to use. She plans on making a rectangular space for Chicken to play, but she plans on using the side of her house as one side of the pen so she will only need to fence the other 3 sides. Write a function to represent the area of the play space she can build for Chicken.



$A(x) = x(160-2x)$, $0 < x < 80$

Also acceptable:



$A(x)=x\left(\frac{160-x}{2}\right)$ $0<x<160$

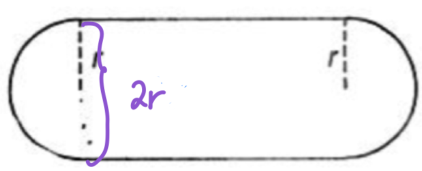
$x>0$ $0<x<$

$160-2x>0$

$-2x>-160$

$x<80$

2. An athletic field is designed as a rectangle with a semicircle at each end. The total perimeter of the field is 600 meters. Express the area of the field as a function in terms of the radius of the semicircles.



$A = \pi r^2 + lw$ need length in terms of r

$A(r) = \pi r^2 + l(2r)$

$A(r) = \pi r^2 + (300-\pi r)(2r)$

$A(r) = \pi r^2 + 600r - 2\pi r^2$, $0<r<\frac{300}{\pi}$

$A(r) = 600r - \pi r^2$, $0<r<\frac{300}{\pi}$

$P = 2\pi r + 2l$

$600 = 2\pi r + 2l$

$\frac{600-2\pi r}{2} = \frac{2l}{2}$

$300-\pi r = l$

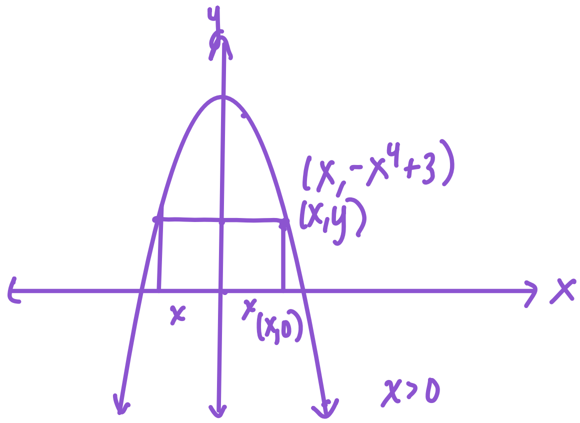
$r>0$

$300-\pi r>0$

$300>\pi r$

$\frac{300}{\pi}>r$

3. Express the area of a rectangle inscribed between the x-axis and $f(x) = -x^4 + 3$.



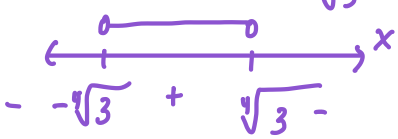
$A = lw$

$A(x) = 2x(-x^4+3)$

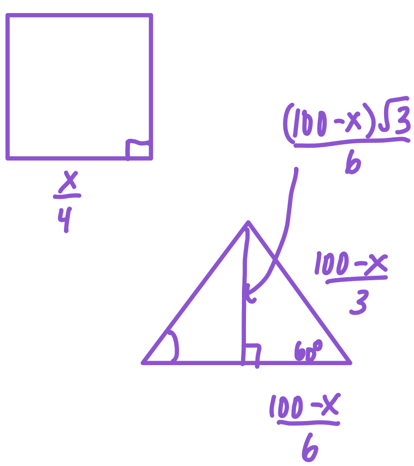
$A(x) = -2x^5 + 6x, \quad 0 < x < \sqrt[4]{3}$

$x > 0$

$3 - x^4 > 0 \qquad -\sqrt[4]{3} < x < \sqrt[4]{3}$



4. We cut a 100 cm piece of wire into two pieces. One piece of length x is bent into the shape of a square and the second piece is bent to make an equilateral triangle. Find a function to model the total combined area of the two figures.



$\frac{100-x}{3}$

$A = s^2 + \frac{1}{2}bh$

$A(x) = \left(\frac{x}{4}\right)^2 + \frac{(100-x)^2\sqrt{3}}{36}$

$A(x) = \frac{x^2}{16} + \frac{(100-x)^2\sqrt{3}}{36}$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}\left(\frac{100-x}{3}\right)\left(\frac{(100-x)\sqrt{3}}{6}\right)$

$x > 0$

$x < 100$

or For Δ part

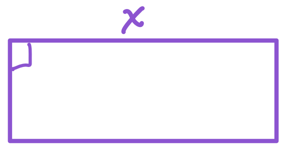
$A_\Delta = \frac{s^2\sqrt{3}}{4}$

$A_\Delta = \frac{\left(\frac{100-x}{3}\right)^2\sqrt{3}}{4}$

$A_\Delta = \frac{\frac{(100-x)^2}{9}\sqrt{3}}{4}$

$A_\Delta = \frac{(100-x)^2\sqrt{3}}{36}$

5. A farmer wants to build a rectangular pen with an area of 250 square meters. Find a function that models the amount of fencing required.



$P = 2l + 2w$

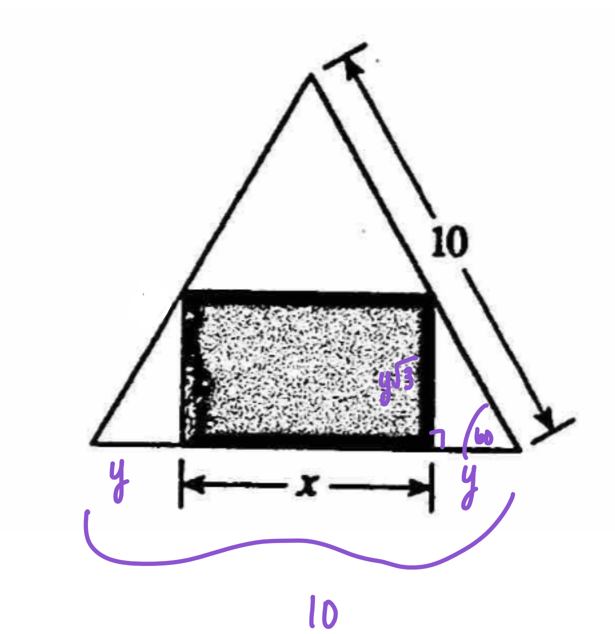
$P(x) = 2x + 2\left(\frac{250}{x}\right), \quad x > 0$

$A = lw$

$250 = xw$

$\frac{250}{x} = w \qquad x > 0$

6. A rectangle is inscribed in an equilateral triangle, as shown in the diagram below, with a perimeter of 30 cm. Express the area of the rectangle as a function of x.



$x > 0$

$x < 10$

$A = lw$ ← need w in terms of x

$A = x \cdot w$

$A(x) = x \cdot \frac{(10-x)\sqrt{3}}{2}, \quad 0 < x < 10$

$x + 2y = 10$

$2y = 10 - x$

$y = \frac{10-x}{2}$

$w = y\sqrt{3}$

$w = \frac{10-x}{2} \cdot \sqrt{3}$

7. A closed rectangular shaped box is x inches wide and 5 times as long. The height of the box is h inches. If the volume of the box is 200 cubic inches, express the surface area of the box as a function of x.

x = width

$5x$ = length

h = height

$$S = 2lw + 2lh + 2wh$$

$$S = 2\cdot 5x\cdot x + 2\cdot 5xh + 2\cdot x\cdot h$$

$$S = 10x^2 + 10xh + 2xh$$

$$S = 10x^2 + 12xh$$ ← need h in terms of x

$$S(x) = 10x^2 + 12\cancel{x}\left(\frac{40}{x^{\cancel{2}}}\right)$$

$$S(x) = 10x^2 + \frac{480}{x}, \quad x > 0$$

$$V = lwh$$

$$200 = 5x\cdot x\cdot h$$

$$200 = 5x^2h$$

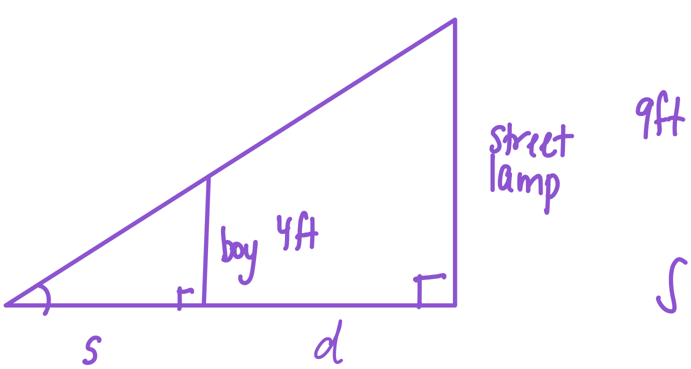
$$\frac{200}{5x^2} = h$$

$$\frac{40}{x^2} = h$$

8. A boy 4 feet tall is standing near a street lamp that is 9 feet tall. Find a function that models the length of the boy's shadow in terms of his distance from the base of the lamp.

$S+x>0$

$x>5$



$$S(d) = \frac{4}{5}d, \quad d>0$$

$$\frac{4}{9} = \frac{s}{s+d}$$

$$4s + 4d = 9s$$

$$4d = 5s$$

$$\frac{4}{5}d = s$$