

$$\begin{array}{r} 2) \quad 1 \quad -5 \quad -5 \quad 23 \quad 10 \\ \quad \quad 2 \quad -6 \quad -22 \quad 2 \\ \hline 1 \quad -3 \quad -11 \quad 1 \quad 8 \end{array}$$

$$\begin{array}{r} 2) \quad 1 \quad -5 \quad -5 \quad 23 \quad 10 \\ \quad \quad -2 \quad 14 \quad -18 \quad -10 \\ \hline 1 \quad -3 \quad 9 \quad 5 \quad 0 \end{array}$$

$$\begin{array}{l} 1-5-5+23+10 \\ 1+5-5-23+10 \end{array}$$

Classwork:

1. Let  $P(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$ . Find the zeros of  $P(x)$ .

possible rat. ans:  $\frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1} = \pm 1, \pm 2, \pm 5, \pm 10$

$$\begin{array}{r} -2) \quad 1 \quad -5 \quad -5 \quad 23 \quad 10 \\ \quad \quad -2 \quad 14 \quad -18 \quad -10 \\ \hline 1 \quad -7 \quad 9 \quad 5 \quad 0 \end{array}$$

$$(x+2)(x-5)(x^2-2x-1) = 0$$

$$x = -2, 5, 1 \pm \sqrt{2} \quad x = \frac{2 \pm \sqrt{4-4(1)(-1)}}{2(1)}$$

$$\begin{array}{r} 5) \quad 1 \quad -7 \quad 9 \quad 5 \quad 0 \\ \quad \quad 5 \quad -10 \quad -5 \\ \hline 1 \quad -2 \quad -1 \quad 0 \end{array}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

2. Factor the polynomial  $P(x) = 2x^3 + x^2 - 13x + 6$

p.r. 2 =  $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

$$\begin{array}{r} 2) \quad 2 \quad 1 \quad -13 \quad 6 \\ \quad \quad 4 \quad 10 \quad -6 \\ \hline 2 \quad 5 \quad -3 \quad 0 \end{array}$$

$$(x-2)(2x^2+5x-3)$$

$$(x-2)(2x-1)(x+3)$$

## Homework 11-29

For 3 - 8, find the complete factorization and all zeros of the following polynomials using the information given.

3.  $P(x) = 2x^5 - 5x^4 + x^3 + 4x^2 - 4x$

$$P(x) = x(2x^4 - 5x^3 + x^2 + 4x - 4)$$

pr 2:  $\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

$$\begin{array}{l} 2-5+1+4-4 \\ -2+5+1-4-4 \end{array}$$

$$x = \frac{3 \pm \sqrt{9-4(2)(2)}}{2(2)} = \frac{3 \pm \sqrt{-7}}{4}$$

$$(x(x+1)(x-2)(2x^2-3x+2))$$

2:  $x, 0, -1, 2, \frac{3 \pm i\sqrt{7}}{4}$

$$\begin{array}{r} -1) \quad 2 \quad -5 \quad 1 \quad 4 \quad -4 \\ \quad \quad -2 \quad 7 \quad -8 \quad 4 \\ \hline 2 \quad -7 \quad 8 \quad -4 \quad 0 \end{array}$$

$$\begin{array}{r} 2) \quad 2 \quad -7 \quad 8 \quad -4 \quad 0 \\ \quad \quad 4 \quad -6 \quad 4 \\ \hline 2 \quad -3 \quad 2 \quad 0 \end{array}$$

-1, -5

4.  $P(x) = x^4 + 6x^3 + 2x^2 - 18x - 15$

$$\begin{array}{r}
 -1 \mid 1 \quad 6 \quad 2 \quad -18 \quad -15 \\
 \quad \quad -1 \quad -5 \quad 3 \quad 15 \\
 \hline
 -5 \mid 1 \quad 5 \quad -3 \quad -15 \quad 0 \\
 \quad \quad -5 \quad 0 \quad 15 \\
 \hline
 \quad \quad 1 \quad 0 \quad -3 \quad 0
 \end{array}$$

CF:  $(x+1)(x+5)(x^2-3)$

Z:  $\{-1, -5, \pm\sqrt{3}\}$

$x^2 - 3$

5.  $P(x) = x^4 - 5x^3 + 3x^2 + 15x - 18$

2, 3

$$\begin{array}{r}
 2 \mid 1 \quad -5 \quad 3 \quad 15 \quad -18 \\
 \quad \quad 2 \quad -6 \quad -6 \quad 18 \\
 \hline
 3 \mid 1 \quad -3 \quad -3 \quad 9 \quad 0 \\
 \quad \quad 3 \quad 0 \quad -9 \\
 \hline
 \quad \quad 1 \quad 0 \quad -3 \quad 0
 \end{array}$$

CF:  $(x-2)(x-3)(x^2-3)$

Z:  $\{2, 3, \pm\sqrt{3}\}$

-3 (mult. of 2)

6.  $P(x) = x^4 + 6x^3 + 7x^2 - 12x - 18$

$$\begin{array}{r}
 -3 \mid 1 \quad 6 \quad 7 \quad -12 \quad -18 \\
 \quad \quad -3 \quad -9 \quad 6 \quad 18 \\
 \hline
 -3 \mid 1 \quad 3 \quad -2 \quad -6 \quad 0 \\
 \quad \quad -3 \quad 0 \quad 6 \\
 \hline
 \quad \quad 1 \quad 0 \quad -2 \quad 0
 \end{array}$$

CF:  $(x+3)^2(x^2-2)$

Z:  $\{-3 \text{ (double)}, \pm\sqrt{2}\}$

0, -1,

7.  $P(x) = x^4 + 3x^3 + 3x^2 + x$

$x(x^3 + 3x^2 + 3x + 1)$

$$\begin{array}{r}
-1 \mid 1 \quad 3 \quad 3 \quad 1 \\
\quad \quad -1 \quad -2 \quad -1 \\
\hline
1 \quad 2 \quad 1 \quad 0
\end{array}$$

$x(x+1)(x^2 + 2x + 1)$

CF  $x(x+1)^3$

Z:  $\{0, -1(\text{triple})\}$

8.  $P(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

$\frac{2}{3}, 4$

prz:  $\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3}$

$$\begin{array}{r}
4 \mid 3 \quad -11 \quad -3 \quad -6 \quad 8 \\
\quad \quad 12 \quad 4 \quad 4 \quad -8 \\
\hline
\end{array}$$

$$\begin{array}{r}
\frac{2}{3} \mid 3 \quad 1 \quad 1 \quad -2 \quad 0 \\
\quad \quad 2 \quad 2 \quad 2 \\
\hline
\end{array}$$

$$\begin{array}{r}
\frac{2}{3} \mid 3 \quad 3 \quad 3 \quad 0 \\
\hline
\end{array}$$

3

CF  $(x-4)(3x-2)(x^2+x+1)$

Z:  $\{4, \frac{2}{3}, \frac{-1 \pm i\sqrt{3}}{2}\}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{-3}}{2}$

17.  $y = x^4 - 5x^2 + 4$

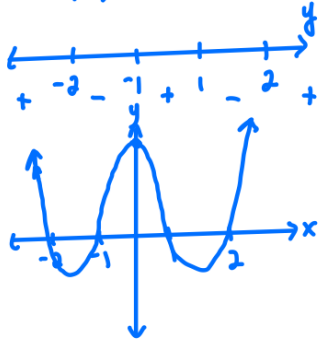
$y = (x^2 - 4)(x^2 - 1)$

$y = (x+2)(x-2)(x+1)(x-1)$

z:  $\pm 2, \pm 1$

y-int:  $(0, 4)$

even degree

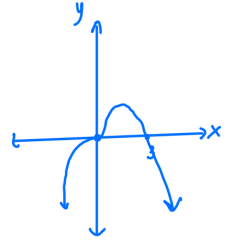
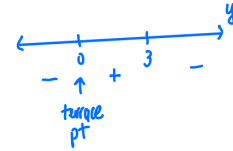


18.  $y = 3x^3 - x^4$

$y = x^3(3-x)$

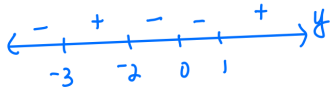
Zeros: 0 (triple), 3

y-int:  $(0, 0)$

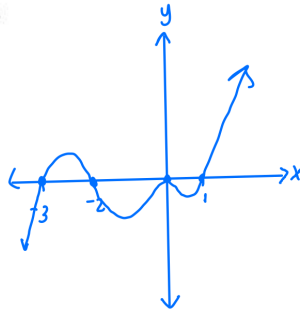


19.  $y = x^2(x-1)(x+2)(x+3)$

Zeros: 0 (double), 1, -2, -3



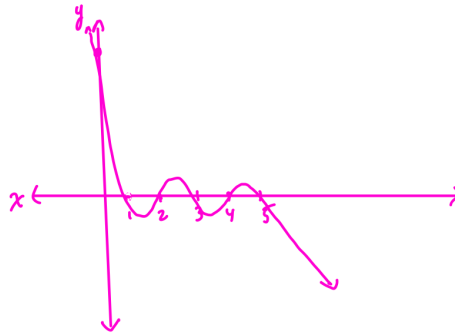
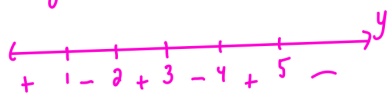
y-int:  $(0, 0)$



20.  $y = (1-x)(2-x)(3-x)(4-x)(5-x)$

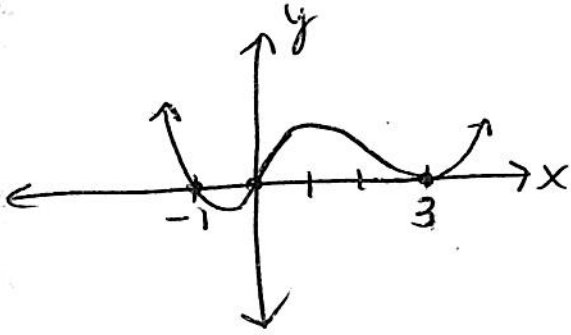
Zeros: 1, 2, 3, 4, 5

y-int:  $(0, 120)$



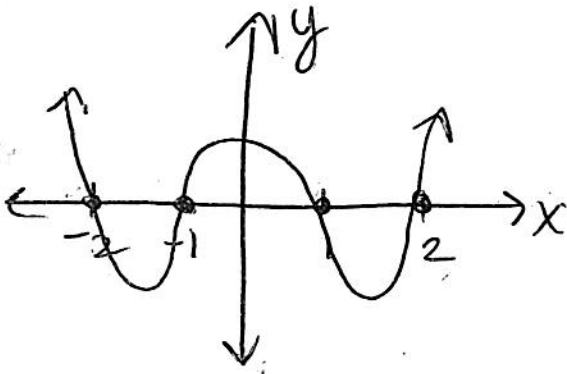
$$(16) \quad y = x(x+1)(x-3)^2$$

zeros:  $-1, 0, 3$  (mult 2)



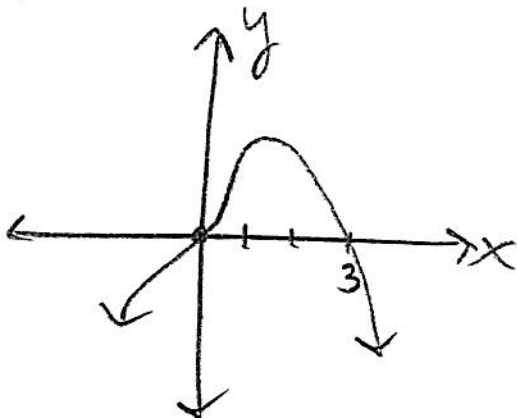
$$(17) \quad y = x^4 - 5x^2 + 4$$
$$y = (x^2 - 4)(x^2 - 1)$$
$$y = (x-2)(x+2)(x-1)(x+1)$$

zeros:  $-2, -1, 1, 2$



$$(18) \quad y = 3x^3 - x^4$$
$$y = x^3(3-x)$$

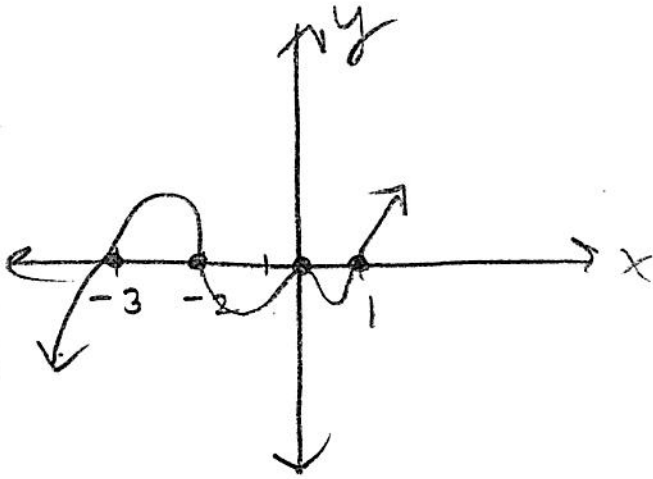
zeros:  $0$  (mult 3),  $3$



19  $y = x^2(x-1)(x+2)(x+3)$

zeros:

-3, -2, 0 (mult 2), 1



20  $y = (1-x)(2-x)(3-x)(4-x)(5-x)$

zeros: 1, 2, 3, 4, 5

