Name: ____ Date: ________
PCH

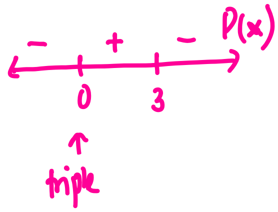
Do Now:

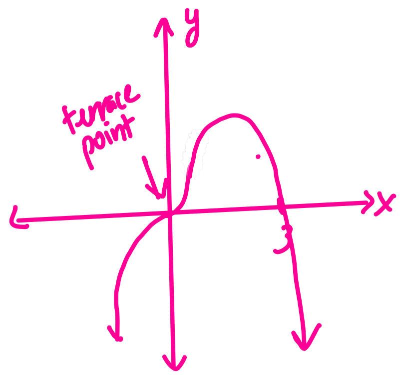
1. Sketch $P(x) = 3x^3 - x^4$. Include and label x and y intercepts with their coordinates and show the correct end behavior of the function.

$P(x) = x^3(3-x)$

$x = 0$ (triple), 3

y-int: $(0,0)$





2. When the function $f(x)$ is divided by $2x+3$, the quotient is $3x^2 - x + 5$ and the remainder is 8. Find the function, $f(x)$, and write the result in standard form.

$f(x) = (2x+3)(3x^2 - x + 5) + 8$

$f(x) = 6x^3 + 7x^2 + 7x + 15 + 8$

$f(x) = 6x^3 + 7x^2 + 7x + 23$

3. On which of the following intervals is $P(x) = x^5 + x^2 - 3x - 4$ guaranteed to have a root? Choose all that apply.

(a) ~~$(-3, 1)$~~ (b) $(1, 2)$ (c) ~~$(-1, -2)$~~

(a) $(-3, 1)$

$P(-3) = (-3)^5 + (-3)^2 - 3(-3) - 4 < 0$

$P(1) = 1 + 1 - 3 - 4 < 0$

(c) $P(-1) = -1 + 1 + 3 - 4 < 0$

$P(-2) = (-2)^5 + (-2)^2 - 3(-2) - 4 < 0$

(b) $(1, 2)$

$P(1) < 0$

$P(2) = 2^5 + 2^2 - 6 - 4 > 0$

Since $P(1) < 0$ and $P(2) > 0$, by the IVT, there has to be an x in $(1,2)$ where $P(x) = 0$.

POLYNOMIAL GRAPH SUMMARY

The following are general statements about the graph of a polynomial function $y = P(x)$.

1. A polynomial function is continuous — i.e. it is defined for all real values of x and can be drawn without lifting the pencil from the paper. Therefore, it has no holes or vertical asymptotes, nor does it have any "jumps" or other gaps.

2. The "turns" taken by the graph are smooth. There are no sharp corners or points.

3. As $|x|$ gets very large, the points of the graph move further and further away from the x-axis. And, there are no horizontal or oblique asymptotes.

The following are statements about the roots of the polynomial equation $P(x) = 0$ with real coefficients and the curve $y = P(x)$.

1. A polynomial equation of degree n has n roots, real and/or nonreal, counting multiplicity.

2. An intersection (not tangency or terrace point) of the curve and the x-axis indicates one real root.

3. A point of tangency located on the x-axis indicates a real root of even multiplicity (2 equal roots, 4 equal roots, etc.).

4. A terrace point located on the x-axis indicates a real root of odd multiplicity greater than 1 (3 equal roots, 5 equal roots, etc.).

5. Non-real roots occur in conjugate pairs.

6. A relative maximum below the x-axis, or a relative minimum above the x-axis, or a terrace point above or below the x-axis indicates one pair of conjugate nonreal roots or a multiplicity of conjugate nonreal roots.
   NOTE: The converse of this statement is false (consider $y = x^4 - 1$).

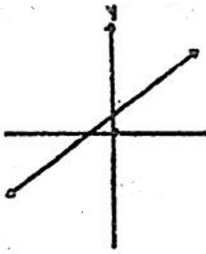
Graphs of Polynomial Functions: End behavior

	Odd degree		Even degree	
Sign of Leading Coefficient	Positive	Negative	Positive	Negative
End behavior	↙ ↗	↖ ↘	↖ ↗	↙ ↘

Pre-Calculus

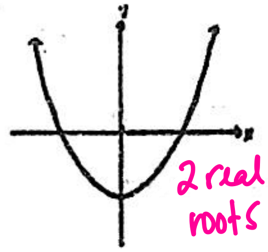
Each graph is a polynomial of degree n. Indicate the number of real roots and the number of non-real roots of $P(x) = 0$. Describe the multiplicity of roots when the multiplicity is greater than 1.

1. $n = 1$



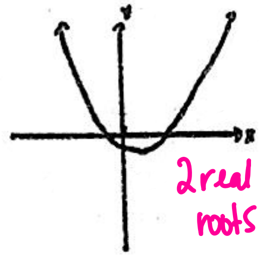
1 real root

2. $n = 2$



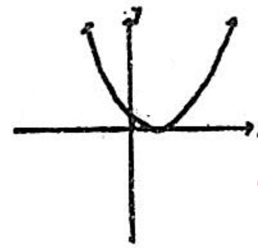
2 real roots

3. $n = 2$



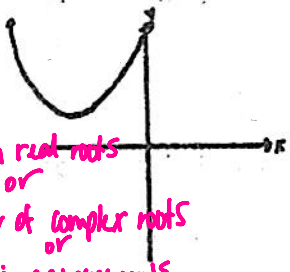
2 real roots

4. $n = 2$



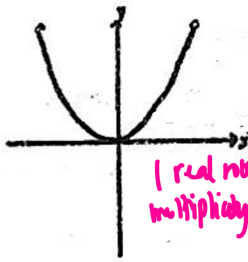
1 real with a multiplicity of 2 or 1 real double root

5. $n = 2$



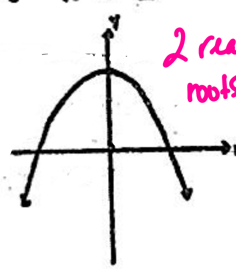
2 non real roots or 1 pair of complex roots or 2 imaginary roots

6. $n = 2$



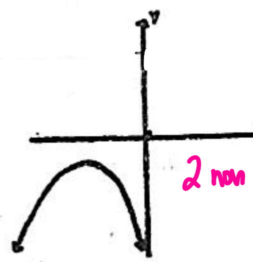
1 real root with multiplicity of 2

7. $n = 2$



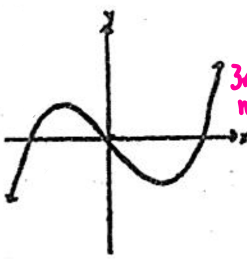
2 real roots

8. $n = 2$



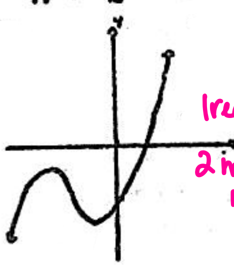
2 non real roots

9. $n = 3$



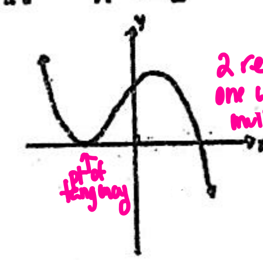
3 real roots

10. $n = 3$



1 real root 2 non real roots

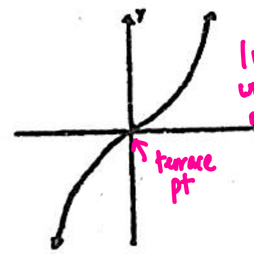
11. $n = 3$



pt of tangency

2 real roots one with a multiplicity of 2

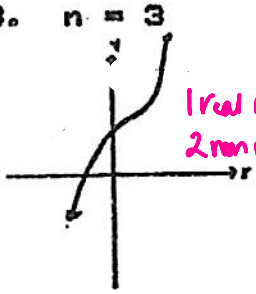
12. $n = 3$



1 real roots with a multiplicity of 3

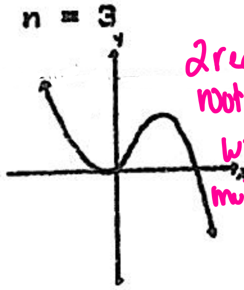
terrace pt

13. $n = 3$



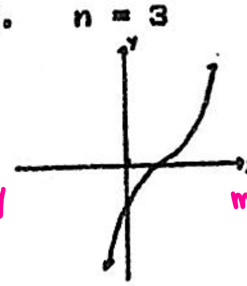
1 real root 2 non real roots

14. $n = 3$



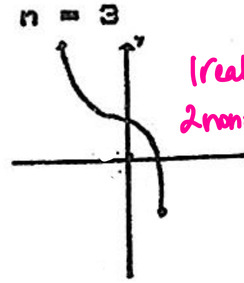
2 real roots, 1 with a multiplicity of 2

15. $n = 3$



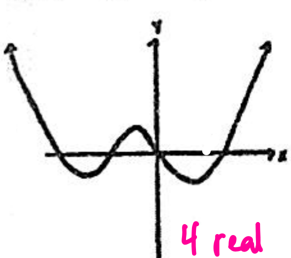
1 real root with mult. of 3

16. $n = 3$



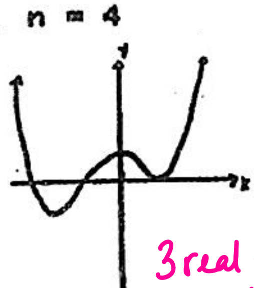
1 real root 2 non real roots

17. $n = 4$



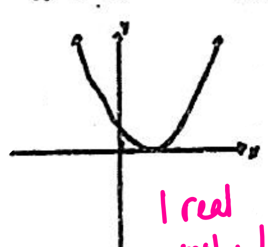
4 real roots

18. $n = 4$



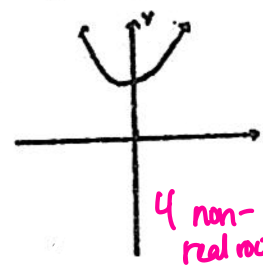
3 real roots one with a mult. of 2

19. $n = 4$



1 real root w/ a mult. of 4

20. $n = 4$



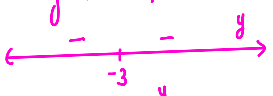
4 non-real roots

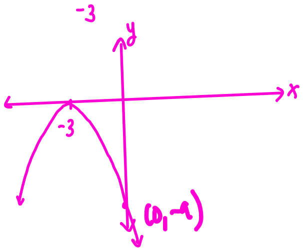
Homework 11-30

3. $y = -(x+3)^2$

zeros: -3 (double)

y-int: $(0,-9)$

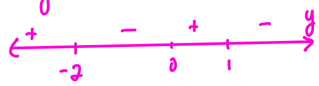


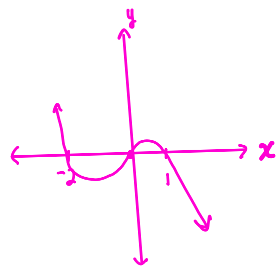


10. $y = -x(x-1)(x+2)$

zeros: $0, 1, -2$

y-int: $(0,0)$





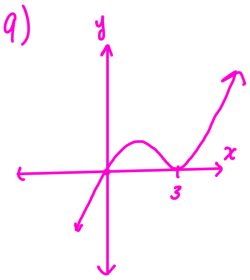
11. $y = x^3 - 6x^2 + 9x$

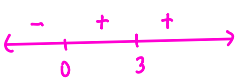
$y = x(x^2-6x+9)$

$y = x(x-3)^2$

zeros: $0, 3$ (double)

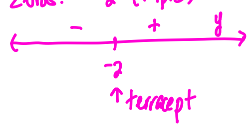
y-int: $(0,0)$

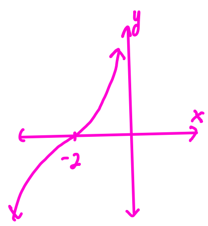




13. $y = (x+2)^3$

zeros: -2 (triple)



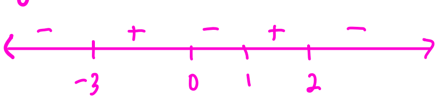


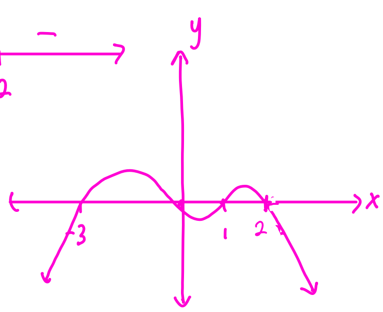
y-int: $(0, 8)$

15. $y = x(1-x)(x-2)(x+3)$

zeros: $0, 1, 2, -3$

y-int: $(0,0)$

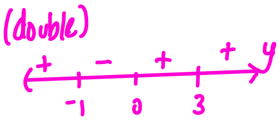


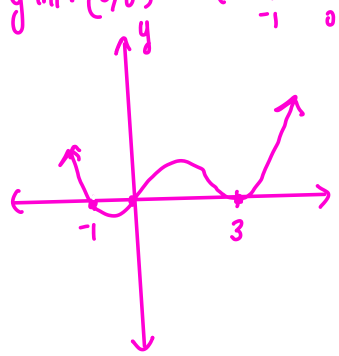


16. $y = x(x+1)(x-3)^2$

zeros: $0, -1, 3$ (double)

y-int: $(0,0)$





23. Find the cubic polynomial whose y-intercept is 9 and whose x-intercepts are 1, 2, and -3.

$\downarrow$
$(0,9)$

$y = a(x-1)(x-2)(x+3)$

$9 = a(0-1)(0-2)(0+3)$

$9 = a(-1)(-2)(3)$

$9 = 6a$

$\frac{3}{2} = a$

$y = \frac{3}{2}(x-1)(x-2)(x+3)$

24. Find the 4th degree polynomial whose graph passes through (0,6) and is tangent to the x-axis at (3,0) and $(-2,0)$.

$\uparrow$ -2 is a double root

$\nearrow$ 3 is a double root

$y = a(x-3)^2(x+2)^2$

$6 = a(0-3)^2(0+2)^2$

$6 = a(9)(4)$

$6 = 36a$

$a = \frac{1}{6}$

$y = \frac{1}{6}(x-3)^2(x+2)^2$