1. Find c and d such that 1 and 2 are roots of the equation $x^3 - 4x^2 + cx + d = 0$.

$1^3 - 4(1)^2 + c(1) + d = 0$

$1 - 4 + c + d = 0$

$-1\,(c + d = 3)$

$2^3 - 4(2)^2 + c(2) + d = 0$

$8 - 16 + 2c + d = 0$

$2c + d = 8$

$$\begin{aligned} -c - d &= -3 \\ 2c + d &= 8 \\ \hline c &= 5 \end{aligned}$$

$c + d = 3$

$5 + d = 3$

$d = -2$

$x^2 - \text{sum}\,x + \text{product} = 0$

2. Determine the value(s) of a such that one root of the equation $x^2 + ax + 12 = 0$ is three times the other.

$a = \pm 8$

product = 12

b = one root 2 or -2

$3b$ = other root 6 or -6

$b \cdot 3b = 12$

$3b^2 = 12$

$b^2 = 4$

$b = \pm 2$

Sum $= 2 + 6 = 8$

or

$-2 - 6 = -8$

③ List all the possible rational roots of

$P(x) = 6x^4 + 10x^3 - 7x^2 + 18$

$$prz = \frac{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18}{\pm 1, \pm 2, \pm 3, \pm 6}$$

$prz = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$

④ Sketch the graph. Include all intercepts and end behavior.

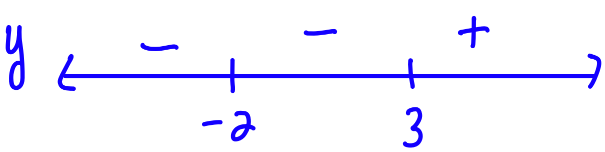
$y = (x-3)(x^2-6x+9)(x^2+4x+4)$

$y = (x-3)(x-3)^2(x+2)^2$

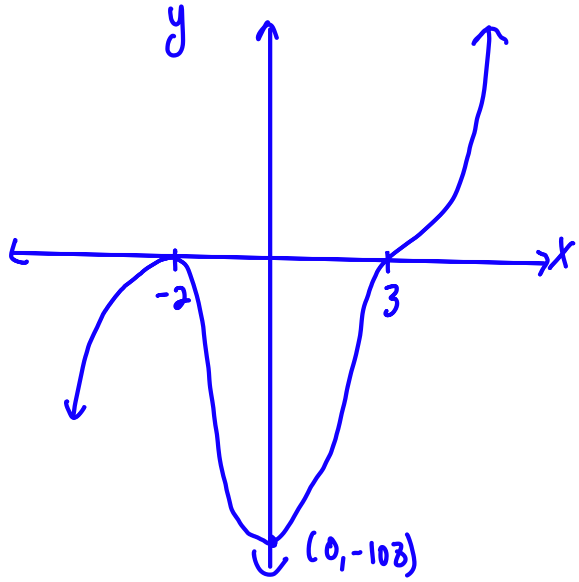
$y = (x-3)^3(x+2)^2$

$x = 3$ (triple), -2 (double)

y-int: $(0, -108)$



$\frac{-27}{4}$



⑤ Is there guaranteed to be a real zero of $P(x) = 3x^3 + 2x^2 - 5x - 4$ between 0 and 1?

$P(0) = -4 < 0$

no

$P(1) = 3 + 2 - 5 - 4 < 0$

⑥ Given $-7 + 3i\sqrt{2}$ is a zero of $P(x) = x^4 + 14x^3 + 66x^2 - 14x - 67$ find the remaining zeros. $\{-7 - 3i\sqrt{2}, \pm 1\}$

another $-7 - 3i\sqrt{2}$

$x^2 - \text{sum}\,x + \text{product}$

sum $= -14$

product $= 49 - 9i^2(2) = 49 + 18 = 67$

$x^2 + 14x + 67$

$$\begin{array}{r|rrrrr}
 & 1 & 14 & 66 & -14 & -67 \\
-14 & & -14 & 0 & 14 & \\
-67 & & & -67 & 0 & 67 \\
\hline
 & 1 & 0 & -1 & 0 & 0
\end{array}$$

$P(x) = (x^2 - 1)(x^2 + 14x + 67)$

$P(x) = (x - 1)(x + 1)(x^2 + 14x + 67)$

$x = 1$ | $x = -1$ | $x = -7 \pm 3i\sqrt{2}$

Turkey Trot Key

Sherwood

The sum of two positive numbers is 60. Find a function that models their product in terms of x, one of the numbers.

$0 < x < 60$

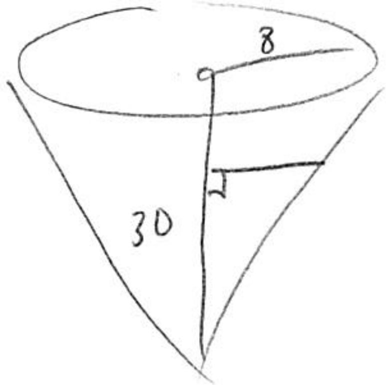
$P(x) = x(60-x)$

$P(x) = 60x - x^2$

$x + y = 60$

$y = 60 - x$

ANS: $A(b) = \dfrac{b\sqrt{36-6b}}{2}$



$\frac{x}{h} = \frac{8}{30}$

$30x = 8h$

$\frac{30x}{8} = h$

$x > 0$

$x < 8$

Sellers

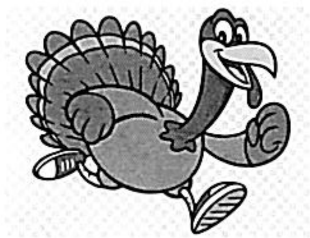
A water tank is in the shape of an inverted right circular cone with altitude 30 feet and radius 8 feet. The tank is filled to a depth of *h* feet. Let *x* be the radius of the circle at the top of the water level. Write a formula for the volume of the water as a function of *x*.

ANS: $P(x) = 60x - x^2$

$V = \frac{1}{3}\pi r^2 h$

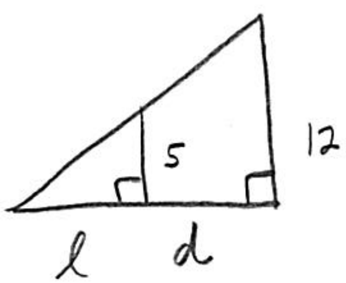
$V(x) = \frac{1}{\cancel{3}}\pi x^2 \left(\frac{\overset{10}{\cancel{30}}x}{8}\right) = \frac{5}{4}\pi x^3$

$0 < x < 8$



Simon

A woman 5 ft tall is standing near a street lamp that is 12 ft tall. Find a functions that models the length of her shadow, *l*, in terms of her distance, *d*, from the base of the lamp.



$\ell(d) = \frac{5}{7}d$

$d > 0$

$\frac{\ell}{5} = \frac{\ell + d}{12}$

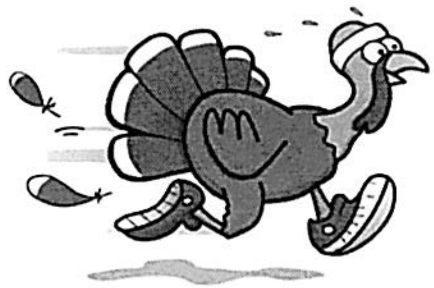
$12\ell = 5\ell + 5d$

$7\ell = 5d$

$\ell = \frac{5}{7}d$

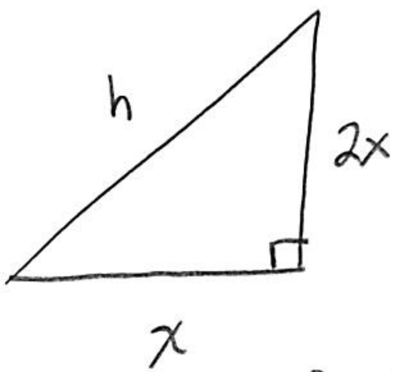
$\ell > 0$

ANS: $V(x) = \frac{5}{4}\pi x^3$



Carman

A right triangle has one leg twice as long as the other. Find a function that models its perimeter in terms of the length x of the shorter leg.



$P(x) = x + 2x + h$

$P(x) = x + 2x + \sqrt{5x^2}$

$P(x) = 3x + \sqrt{5x^2}$, $x > 0$

$= 3x + x\sqrt{5}$ ANS: $l(d) = \frac{5}{7}d$

$h^2 = (2x)^2 + x^2$

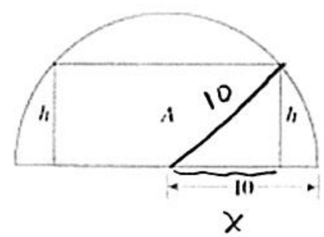
$h^2 = 5x^2$

$h = \pm\sqrt{5x^2}$

Loughran

A rectangle is inscribed in a semicircle of radius 10, as shown in the figure. Find a function that models the area of the rectangle in terms of its height h.

$0 < h < 10$



$A = lw$

$A(h) = l \cdot h$

$A(h) = 2\sqrt{100-h^2} \cdot h$

$A(h) = 2h\sqrt{100-h^2}$

ANS: $P(x) = 3x + x\sqrt{5}$

$x^2 + h^2 = 10^2$

$x^2 + h^2 = 100$

$x^2 = 100 - h^2$

$x = \pm\sqrt{100-h^2}$

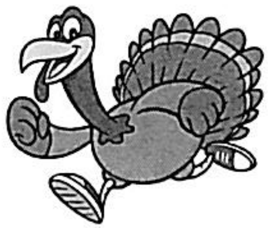
$h > 0$

$100 - h^2 > 0$

$(10-h)(10+h) > 0$

$-10 \quad 10$

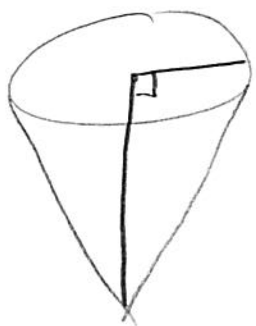
$-10 < h < 10$



Barnett

The volume of a cone is 100 cubic inches. Find a function that models the height *h* of the cone in terms of its radius *r*.

$h(r) = \frac{300}{\pi r^2} \quad r > 0$



ANS: $A(h) = 2h\sqrt{100 - h^2}$

$V = \frac{1}{3}\pi r^2 h$

$100 = \frac{1}{3}\pi r^2 h$

$300 = \pi r^2 h$

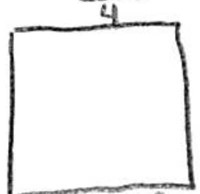
$h = \frac{300}{\pi r^2}$

Windwer

A wire 10 cm long is cut into two pieces, one of length x and the other of length $10 - x$. Each is bent into the shape of a square. Find a function that models the total area enclosed by the two squares.

$x > 0$
$x < 10$

$A_{sq}(x) = \left(\frac{x}{4}\right)^2$



$A(x) = \left(\frac{10-x}{4}\right)^2$

$$A(x) = \frac{x^2}{16} + \frac{(10-x)^2}{16}$$

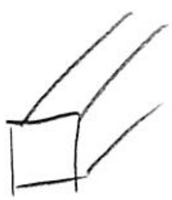
$$A(x) = \frac{x^2 + 100 - 20x + x^2}{16} = \frac{2x^2 - 20x + 100}{16} = \frac{x^2}{8} - \frac{5x}{4} + \frac{25}{4}$$

$0 < x < 10$

ANS: $h(r) = \frac{300}{\pi r^2}$

Lee

An ***open*** box with a square base is to have a volume of 12 cubic feet. Find a function that models the surface area of the box.



$V = lwh$

$12 = lwh$

$12 = x \cdot x \cdot h$

$\frac{12}{x^2} = h$

$x > 0$

ANS: $A(x) = \frac{1}{8}x^2 - \frac{5}{4}x + \frac{25}{4}$

$S = x^2 + 2xh + 2xh$

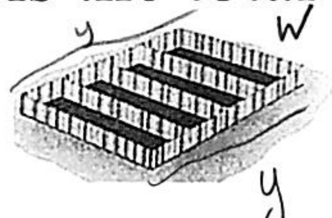
$S = x^2 + 4xh$

$S(x) = x^2 + 4\cancel{x}\left(\frac{12}{x^{\cancel{2}}}\right)$

$S(x) = x^2 + \frac{48}{x}$

Jacknis

A rancher with 750 ft of fencing wants to enclose a rectangular area then divide it into four pens with fencing parallel to one side of the rectangle (see figure). Find a function that models the total area of the four pens. in terms of w



$w > 0$

$150 - w > 0$

$150 > w$

$5w + 2y = 750$

$5w = 750 - 2y$

$2y = 750 - 5w$

$y = \frac{750-5w}{2} = 375 - \frac{5}{2}w$

$A(x) = w\left(\frac{750-5w}{2}\right)$ factored out a 5

$A(x) = \frac{5w(150-w)}{2}$

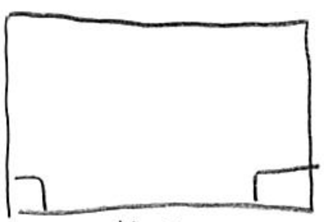
$0 < w < 150$

ANS: $A(x) = \frac{48}{x} + x^2$

Stack

A sheet of cardboard 3 feet by 4 feet will be made into a box by cutting equal squares, of length x, from each corner and folding up the four edges. Express the volume of this box as a function of x.

ANS: $A(w) = \frac{5}{2}w(150 - w)$



$V(x) = x(3-2x)(4-2x)$

$V(x) = (3x-2x^2)(4-2x)$ $\quad 0 < x < \frac{3}{2}$

$x > 0$

$4-2x > 0$, $4 > 2x$, $2 > x$

$3-2x > 0$, $3 > 2x$, $\frac{3}{2} > x$

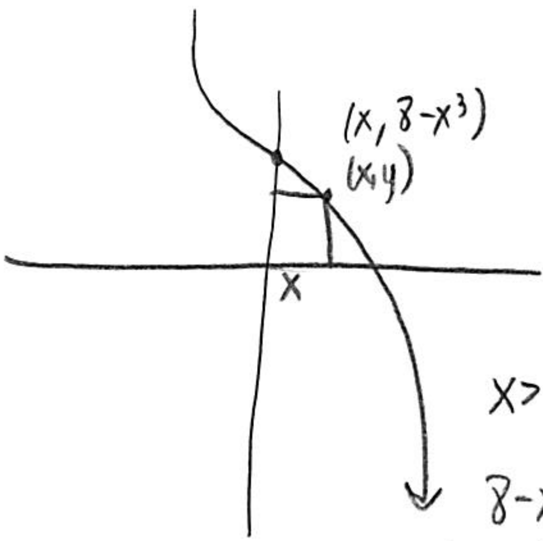
$V(x) = 12x - 8x^2 - 6x^2 + 4x^3$

$V(x) = 4x^3 - 14x^2 + 12x$

Ciavarella

A rectangle is inscribed between the x-axis, the y-axis and the graph of $y=8-x^3$. Write the area of the rectangle as a function of x.

$y=-x^3+8$



$A(x)=x(8-x^3)$
$A(x)=8x-x^4$

$0<x<2$

$x>0$

$8-x^3>0$

$(2-x)$

ANS: $V(x)=4x^3-14x^2+12x$

Edelman

A manufacturer makes a metal can that holds 1L or 1000 cubic centimeters. Write a formula for the surface area of the can in terms of its radius.

$1000 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

$r > 0$

$S = 2\pi r^2 + 2\pi r h$

$S(r) = 2\pi r^2 + 2\cancel{\pi r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$

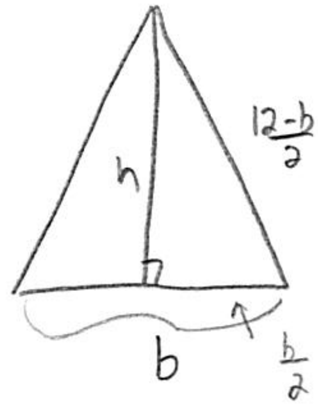
$r > 0$

ANS: $A(x) = -x^4 + 8x$

$S(r) = 2\pi r^2 + \frac{2000}{r}$

Callahan

$\frac{12-b}{2}$



$h^2+\left(\frac{b}{2}\right)^2=\left(\frac{12-b}{2}\right)^2$

$h^2+\frac{b^2}{4}=\frac{b^2-24b+144}{4}$

$h^2=\frac{\cancel{b^2}-24b+144-\cancel{b^2}}{4}$

$h^2=\frac{144-24b}{4}$

$h^2=36-6b$

$h=\pm\sqrt{36-6b}$

An isosceles triangle has a perimeter of 12 cm. Find a function that models its area in terms of the length of its base *b*.

$A=\frac{1}{2}bh$

$0<b<6$

$A(b)=\frac{1}{2}b\left(\sqrt{36-6b}\right)$

$36-6b>0$

$36>6b$

$6>b$

ANS: $S(r)=2\pi r^2+\frac{2000}{r}$