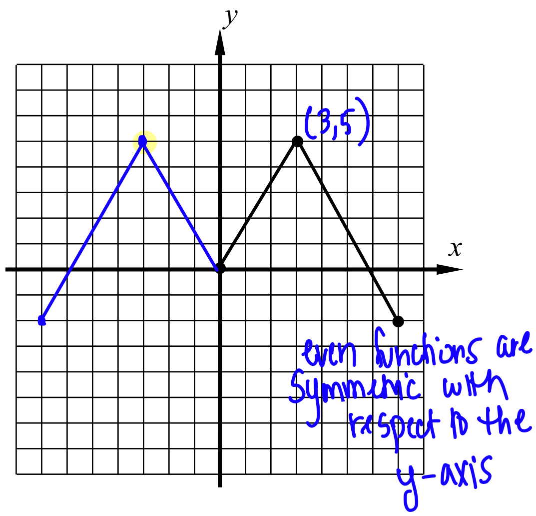
Name: ______________________________________ Date: ___________________

PCH Even and Odd Functions

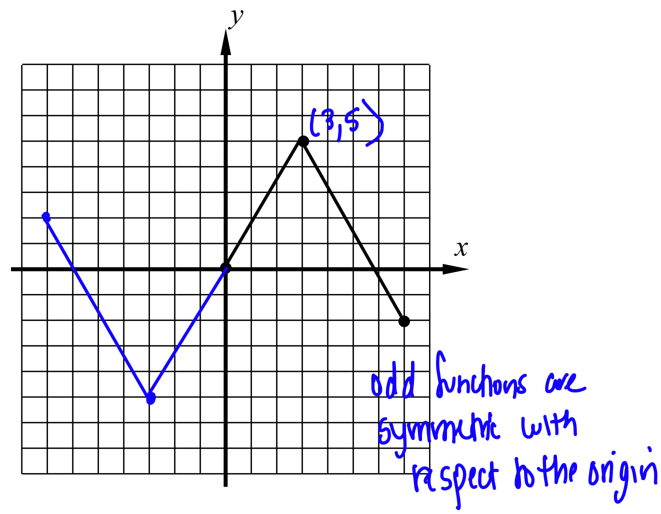
Do Now:

Consider the **partial graph** of the function $f(x)$ shown twice below. Sketch the other half of the function if in (a) $f(x)$ is **even** and in (b) $f(x)$ is **odd**. The three coordinate pairs are listed to help you plot.

(a) **even**



(b) **odd**



Let f be a function.

f is even if $f(-x)=f(x)$ for all x in the domain of f

f is odd if $f(-x)=-f(x)$ for all x in the domain of f

The graph of an even function is symmetric with respect to the y-axis.

The graph of an odd function is symmetric with respect to the origin.

Examples:

Determine whether the functions are even, odd, or neither even nor odd.

1. $f(x)=x^5+x$

$f(-x)=(-x)^5+(-x)$

$f(-x)=-x^5-x$

$\uparrow -f(x)$

odd

2. $g(x)=1-x^4$

$g(-x)=1-(-x)^4$

$g(-x)=1-x^4$

$\downarrow g(x)$

even

3. $h(x)=2x-x^2$

$h(-x)=-2x-x^2$

$\uparrow$ not $h(x)$ or $-h(x)$

so it is neither

4. $f(x)=3x^3+2x^2+1$

$f(-x)=-3x^3+2x^2+1$

neither

5. $g(x)=x+\frac{1}{x}$

$g(-x)=-x-\frac{1}{x}$

odd

6. $h(x)=x^4-4x^2$

$h(-x)=x^4-4x^2$

even

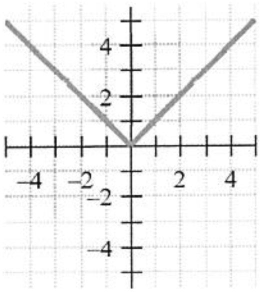
Exercises

1. If a function is even, its graph is symmetric with respect to the y-axis.
   This also means that $f(-x) =$ $f(x)$

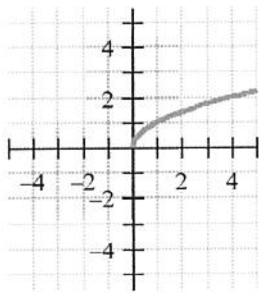
2. If a function is odd, its graph is symmetric with respect to the origin.
   This also means that $f(-x) =$ $-f(x)$

Determine whether each function graphed is even, odd, or neither

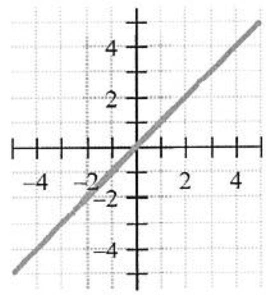
E 3.



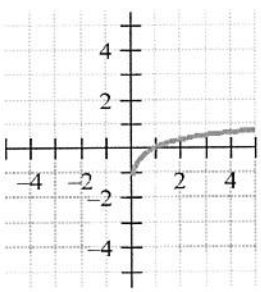
N 4.



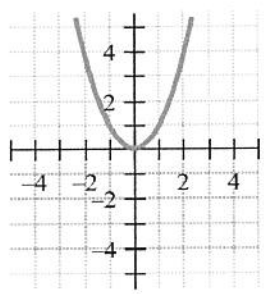
O 5.



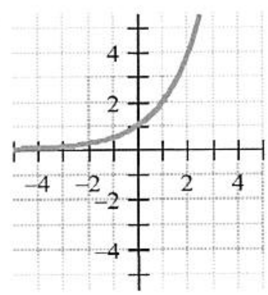
N 6.



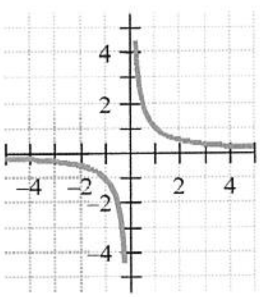
E 7.



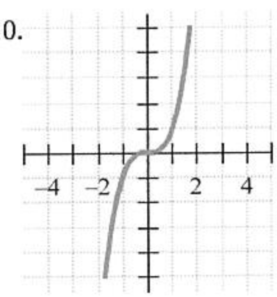
N 8.



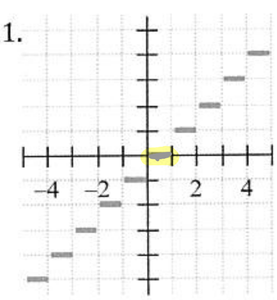
O 9.



O 10.



N 11.



Determine algebraically whether each of the following functions is even, odd or neither.

12. $f(x) = 4x + 5$

$f(-x) = -4x + 5$ neither

13. $f(x) = x^3 - x$

$f(-x) = -x^3 + x$ odd

14. $f(x)=x^2-6$

$f(-x)=x^2-6$

even

15. $f(x)=x^3-x-2$

$f(-x)=-x^3+x-2$

neither

16. $f(x)=\dfrac{x^4-x}{x^5-x}$

$f(-x)=\dfrac{x^4+x}{-x^5+x}$

neither

* for a fraction just the numerator or denominator has to be multiplied by -1 to make it odd
* if both the numerator and the denominator are multiplied by -1, the function even

17. $f(x)=\dfrac{x^3-x}{x^5}$

$f(-x)=\dfrac{-x^3+x}{-x^5}=\dfrac{+(x^3-x)}{+x^5}$

even

18. $f(x)=(x-4)^2$

$f(-x)=(-x-4)^2$

$=x^2+8x+16$

neither

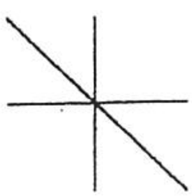
19. $f(x)=x^4-x^2+4$

$f(-x)=x^4-x^2+4$

even

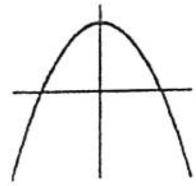
More Practice

1. Indicate which of the following functions are even, which are odd, and which are neither.



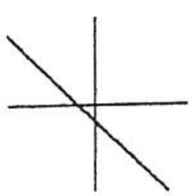
Graph (a)

ODD



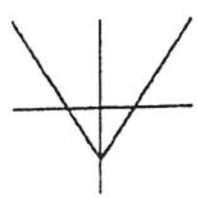
Graph (b)

EVEN



Graph (c)

NEITHER



Graph (d)

EVEN

2. Algebraically, determine whether each function is odd, even, or neither.

E a) $f(x)=3x^4-5x^2+17$

E b) $f(x)=|x|$

O c) $f(x)=12x^7+6x^3-2x$

N d) $f(x)=4x^3-7$

N e) $f(x)=x^2+2x+2$

O f) $f(x)=\dfrac{x^2-5}{2x^3+x}$

$f(-x)=\dfrac{x^2-5}{-2x^3-x}$

Even More Practice

1. For each of the following functions, *use the definitions above* to determine whether each is an even function, an odd function, both, or neither. Also, state the type of symmetry exhibited by each even and odd function.

a) $y = x^4$ even

b) $y = x^3 - 4x$ odd

c) $y = x^2 + x$ neither

d) $y = 0$ both

e) $y = 7$ even

f) $y = 7x$ odd

g) $y = 7x + 1$ neither

h) $y = 3x^2 - 5$ even

i) $y = 3x^3 - 5$ neither

j) $y = 3x^3 - 5x$ odd

k) $y = \frac{1}{x}$ odd

l) $y = \frac{1}{1 + x^2}$ even

m) $y = \frac{x^2 + 6}{x}$ odd

n) $y = \frac{x}{x^3 + 1}$ neither

o) $y = |x|$ even

p) $y = |x| + 2$ even

q) $y = |x + 2|$ neither

r) $y = \sqrt{4 - x^2}$ even

s) $y = \sqrt{x}$ neither

t) $y = \sqrt[3]{x}$ odd

u) $\{(-2,0), (2,0)\}$ both

2. Sketch the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$. Determine which of these are even and odd.

↑ odd ↑ even ↑ odd on $\{x \mid x \neq \frac{k\pi}{2}, k \in \text{odd } \mathbb{Z}\}$