Do Now : (Also make a sketch)

Polynomial:

$P(x) = x^3 + 3x^2 + 3x + 1$

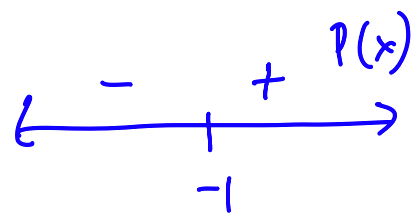
Possible Rational Zeros:

± 1

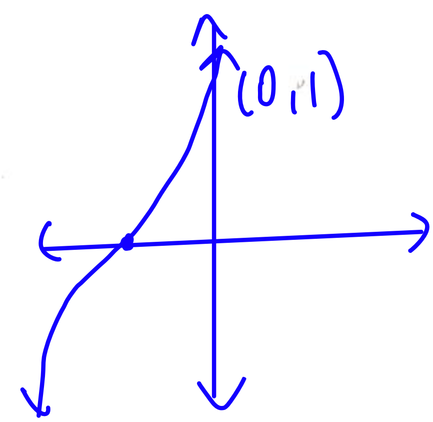
$P(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$(x+1)(x^2+2x+1)$



Sketch :



Complete Factorization:

$P(x) = (x+1)^3$

Complete Solution Set:

-1 (triple)

Check:

$(x+1)(x^2+2x+1) = x^3 + 3x^2 + 3x + 1$

Name:____ Date:_______________
PCH: Reducible Rational Functions Ms. Loughran

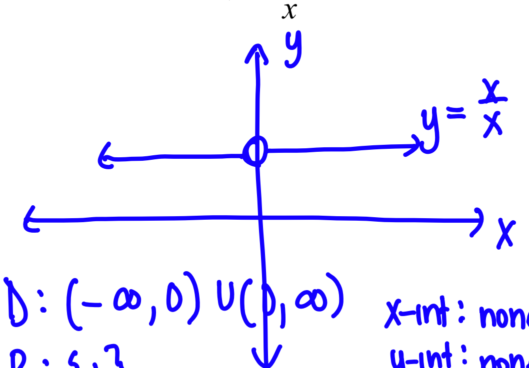
Undefined: $\frac{a}{b}$ where $b=0$ and $a \neq 0$

Indeterminate: $\frac{a}{b}$ where $b=0$ and $a=0$

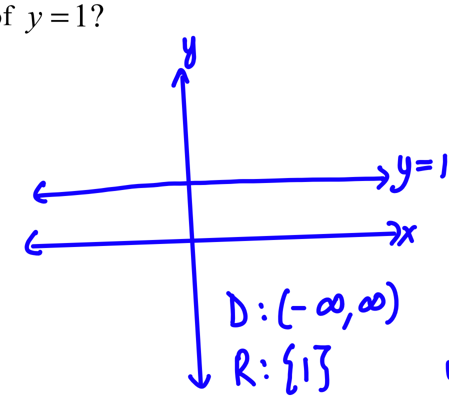
A rational function that is indeterminate for a value of x is *reducible*. A "hole" occurs at the value(s) of x which make the given function indeterminate and the reduced fraction defined.

$y=1$ with a hole at $(0,1)$

Is the graph of $y=\frac{x}{x}$ the same as the graph of $y=1$?



D: $(-\infty,0)\cup(0,\infty)$
R: $\{1\}$
x-int: none
y-int: none



D: $(-\infty,\infty)$
R: $\{1\}$
x-int: none
y-int: $(0,1)$

If a function is reducible use the reduced function when finding the intercepts.

$x+2=0$

Sketch the graph of each of the following. State the domain, range, and any intercepts.

1. $y=\frac{x^2-4}{x+2} = \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

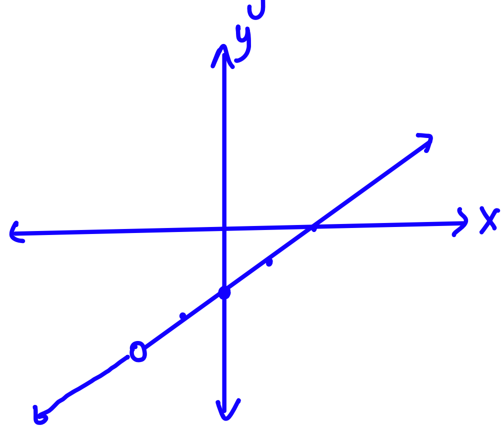
hole: $(-2,-4)$

plug -2 into the RF

Use RF

x-int: (let $y=0$)
$(2,0)$

Reduced Function: $y=x-2$



y-int:
$(0,-2)$

D: $(-\infty,-2)\cup(-2,\infty)$
or
$\{x \mid x \neq -2\}$

R: $(-\infty,-4)\cup(-4,\infty)$
or
$\{y \mid y \neq -4\}$

2. $y = \dfrac{x^2 - 5x + 6}{3 - x}$

$x-3=0$
or
$3-x=0$

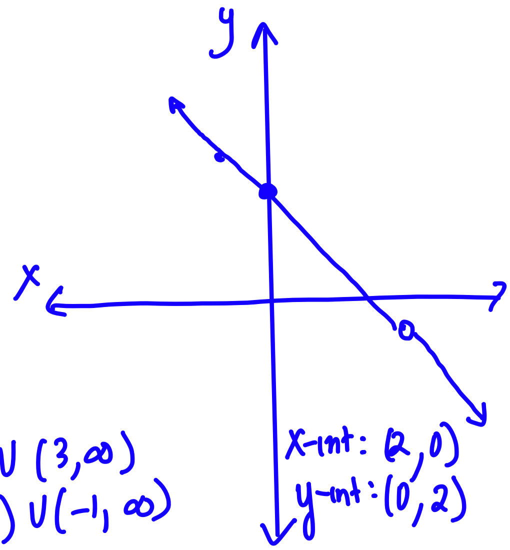
$$y = \frac{\cancel{(x-3)}^{\,-1}(x-2)}{\cancel{3-x}}$$

hole: $(3, -1)$

plug 3 into the RF

RF $y = -(x-2) = -x+2$

D: $(-\infty, 3) \cup (3, \infty)$
R: $(-\infty, -1) \cup (-1, \infty)$



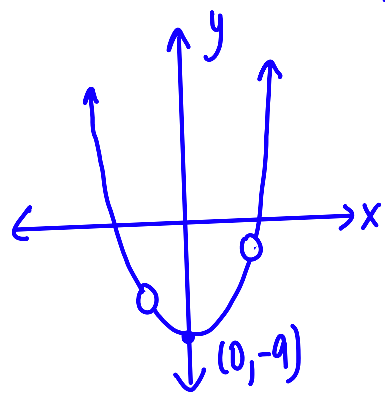
x-int: $(2, 0)$
y-int: $(0, 2)$

3. $y = \dfrac{\cancel{(x+1)}(x+3)(x-3)\cancel{(x-2)}}{\cancel{(x+1)}\cancel{(x-2)}}$

holes: $(-1, -8)$
$(2, -5)$

D: $\{x \mid x \neq -1, 2\}$
R: $[-9, \infty)$
or
$\{y \mid y \geq -9\}$

RF: $y = (x+3)(x-3)$
$y = x^2 - 9$



y-int: $(0, -9)$
x-int: $(\pm 3, 0)$

4. $y = \dfrac{x^3 - 1}{x - 1}$

$$y = \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$$

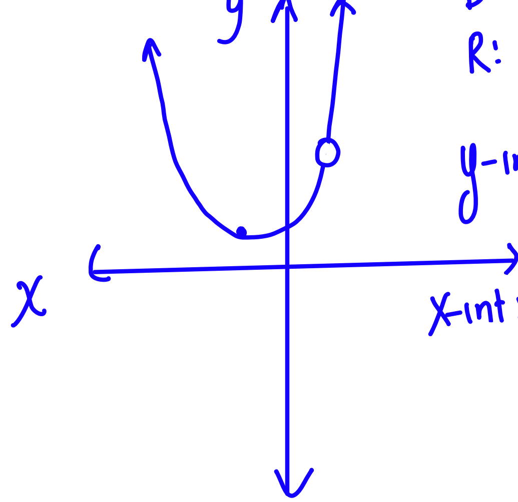
hole: $(1, 3)$

D: $\{x \mid x \neq 1\}$
R: $[\frac{3}{4}, \infty)$

RF $y = x^2 + x + 1$

$y = x^2 + x + \frac{1}{4} - \frac{1}{4} + 1$

$y = (x + \frac{1}{2})^2 + \frac{3}{4}$



y-int: $(0, 1)$

x-int: none

Practice

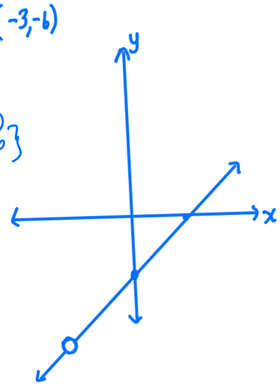
Sketch the graph of each of the following. State the domain, range, and any intercepts.

1. $y = \dfrac{x^2 - 9}{x + 3}$

$y = \dfrac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} = x - 3$

hole: $(-3, -6)$

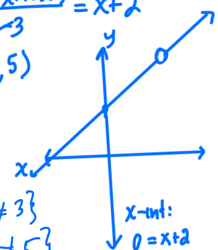
D: $\{x \mid x \neq -3\}$
R: $\{y \mid y \neq -6\}$
x-int: $(3, 0)$
y-int: $(0, -3)$



2. $y = \dfrac{x^2 - x - 6}{x - 3}$

$y = \dfrac{\cancel{(x-3)}(x+2)}{\cancel{x-3}} = x + 2$

hole: $(3, 5)$



D: $\{x \mid x \neq 3\}$
R: $\{y \mid y \neq 5\}$
x-int: $(-2, 0)$
y-int: $(0, 2)$

x-int:
$0 = x + 2$
$-2 = x$