Name:_____ Date:_____

PCH: Oblique Asymptotes Ms. Loughran

Do Now:

1. Find the vertical asymptote(s) of the function $y=\frac{x+6}{x^2-36}$ $=\frac{1}{x+6}$ $\frac{(x+6)}{(x+6)(x-6)}$ VA: $x-6=0$, $x=6$

2. Find the horizontal asymptote of the function $y=\frac{x^2+2x+1}{x+1}$ none

3. Is there a hole in the graph of $y=\frac{x^2+9}{x+3}$? no, it is reducible

4. What is the domain of the function $y=\frac{x^2-x-12}{x-4}$? $\frac{(x-4)(x+3)}{x-4}$ $y=x+3$ hole $(4,7)$ $(-\infty,4)\cup(4,\infty)$ or $\{x \mid x\neq 4\}$

5. Are there any x- or y- intercepts for the graph of $y=\frac{3x^2+x-2}{x+1}$? If so, state them. $\frac{(3x-2)(x+1)}{x+1}$ hole: $(-1,-5)$ $y=3x-2$ x-int: $(\frac{2}{3},0)$ y-int: $(0,-2)$

When the end behavior of a rational function is not horizontal (meaning there is no horizontal asymptote), it is oblique.

Recall: In what situation is there no horizontal asymptote for a rational function?

Happens when the degree of the numerator > the degree of the denominator.

To find oblique asymptotes:

1. **reduce the function if possible**
2. **divide the numerator by the denominator using long or synthetic division**
3. **the oblique asymptote is y = the quotient** (ignore remainder)

1. Find the oblique asymptote of $y=\frac{x^2-3x+5}{x+2}$

$$\begin{array}{r|rrr} -2 & 1 & -3 & 5 \\ & & -2 & 10 \\ \hline & 1 & -5 & 15 \end{array}$$ ← remainder

Oblique Asymptote: $y=x-5$

2. Find the oblique asymptote of $y = \frac{x^2}{x+1}$.

OA

$$\begin{array}{r|rr} -1 & 1 & 0 & 0 \\ & & -1 & 1 \\ \hline & 1 & -1 & (1) \end{array}$$ remainder

OA: $y = x - 1$

3. Find the oblique asymptote for $y = \frac{x^2 - 4}{x}$

can't use synthetic division

$$\begin{array}{r} x \\ x \overline{)\, x^2 \quad -4} \\ \underline{x^2} \\ (-4) \end{array}$$ remainder

OA: $y = x$

4. Find the oblique asymptote of $y = \frac{x^2 - 1}{-x + 3}$

$-(x-3)$

$$\begin{array}{r|rr} 3 & 1 & 0 & -1 \\ & & 3 & 9 \\ \hline & 1 & 3 & (8) \end{array}$$ remainder

$\div -1$

OA: $y = -x - 3$

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PCH: Practice with Vertical, Horizontal and Oblique Asymptotes Ms. Loughran

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote Does graph intersect HA?	Oblique Asymptote Does graph intersect the OA?	*x*-intercept(s)	*y*-intercept
① $y=\frac{x-5}{x^2-4x-5}$ RF $=\frac{1}{x+1}$ $(x-5)(x+1)$	$(5,\frac{1}{6})$	$x+1=0$ $x=-1$	$y=0$ $\frac{1}{x+1}\neq 0$ NO	NO	none	$(0,1)$
② $y=\frac{2x+1}{x^2}$						
③ $y=\frac{x-5}{x^2+1}$						
④ $y=\frac{2x}{x^2-x-6}$						
⑤ $y=\frac{-3x^2+2}{x-1}$	no	$x=1$	no	$y=-3x-3$ no	$\frac{-3x^2+2}{x-1}=0$ $-3x^2=-2$ $x^2=\frac{2}{3}$ $x=\pm\sqrt{\frac{2}{3}}$ $(\pm\sqrt{\frac{2}{3}},0)$	$(0,-2)$
⑥ $y=\frac{2x^3-17x^2-8x-9}{3-x^2}$						

⑤ 1| -3 0 2
-3 -3
-3 -3 -1

$\frac{-3x^2+2}{x-1}=\frac{-3x-3}{1}$

$-3x^2+2=-3x^2-3x+3x+3$

$-3x^2+2=-3x^2+3$

$2 \neq 3$

Homework 12-11

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote Does graph intersect HA?	x-intercept(s) let $y=0$	y-intercept let $x=0$
(1) $y=\frac{1-x}{x+3}$	no	$x+3=0$ $x=-3$	$y=\frac{-1}{1}=-1$ $\frac{1-x}{x+3}=-1$ $-x-3=1-x$ $-3 \neq 1$ Doesn't intersect HA	$0=\frac{1-x}{x+3}$ $0=1-x$ $x=1$ $(1,0)$	$y=\frac{1-0}{0+3}=\frac{1}{3}$ $(0,\frac{1}{3})$
(2) $y=\frac{x-2}{x^2-4}$ $(x+2)(x-2)$ $y=\frac{1}{x+2}$	$(2,\frac{1}{4})$	$x+2=0$ $x=-2$	$y=0$ $\frac{1}{x+2}=0$ $1 \neq 0$ No	$0=\frac{1}{x+2}$ $0 \neq 1$ none	$y=\frac{1}{0+2}$ $(0,\frac{1}{2})$
(3) $y=\frac{x^2-x-20}{x+4}$ $(x-5)(x+4)$ $y=x-5$	$(-4,-9)$	none	none	$0=x-5$ $5=x$ $(5,0)$	$y=0-5$ $y=-5$ $(0,-5)$
(4) $y=\frac{x^2-x-20}{x+1}$ $(x-5)(x+4)$	no	$x+1=0$ $x=-1$	none	$0=\frac{(x-5)(x+4)}{x+1}$ $0=(x-5)(x+4)$ $(5,0),(-4,0)$	$y=\frac{-20}{1}$ $(0,-20)$
(5) $y=\frac{2x^2}{x^3+x}=\frac{2x^2}{x^2+1}$ $x(x^2+1)$	$(0,0)$	$x^2+1=0$ $x^2=-1$ $x=\pm i$ none	$y=2$ $\frac{2x^2}{x^2+1}=2$ no	$0=\frac{2x^2}{x^2+1}$ $0=2x^2$ hole $0=x^2$ $(0,0)$ $0=x$ (no)	no
(6) $y=\frac{x-1}{x^2-4}$ $(x-2)(x+2)$	no	$(x-2)(x+2)=0$ $x=\pm 2$	$y=0$ $\frac{x-1}{(x-2)(x+2)}=0$ $x-1=0$ $x=1$ $(1,0)$ intersect the HA	$0=\frac{x-1}{x^2-4}$ $0=x-1$ $x=1$ $(1,0)$	$y=\frac{0-1}{0^2-4}=\frac{1}{4}$ $(0,\frac{1}{4})$

$2x^2=2x^2+2$

$0 \neq 2$