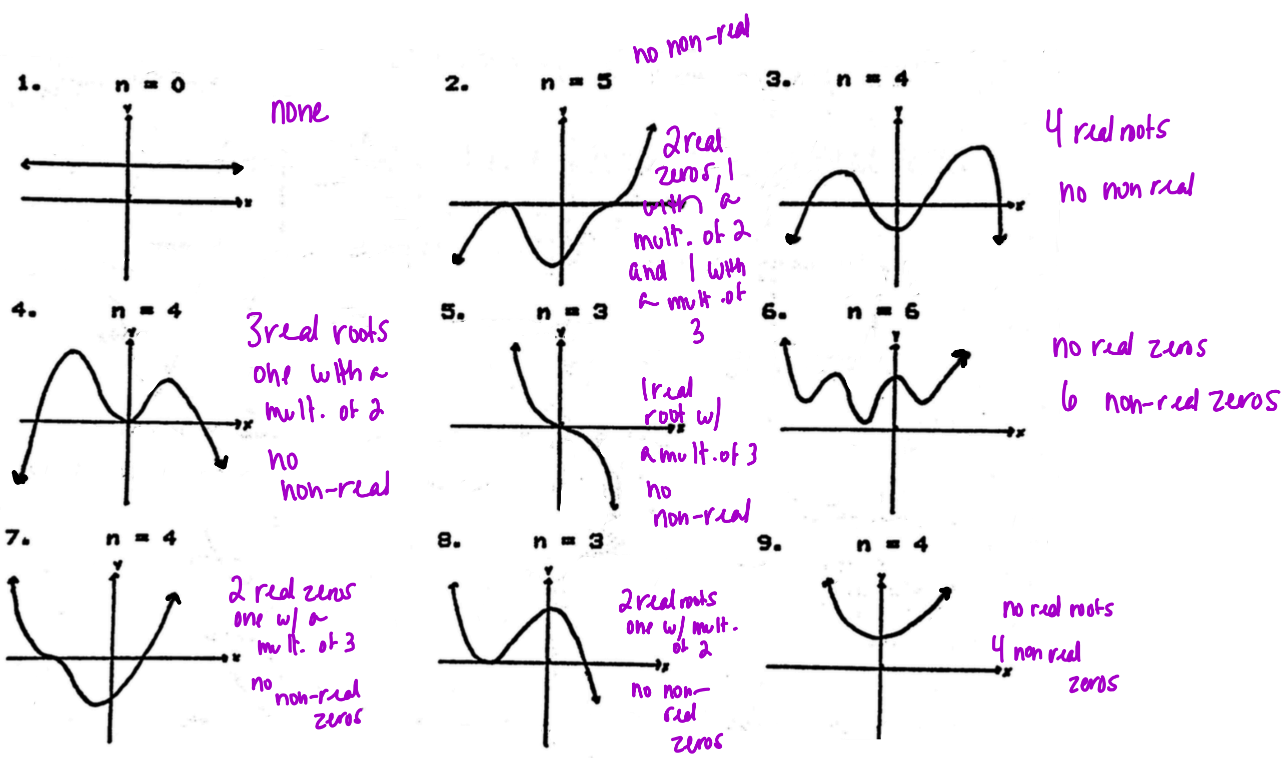
Name: ________________________________ Date: ________

PCH: Graphs of Polynomials Ms. Loughran

Do Now:

1. For each of the following graphs, given the degree of the polynomial, determine:
   (a) the number of distinct real zeros.
   (b) the multiplicity of any real zeros.
   (c) the number of non-real zeros.



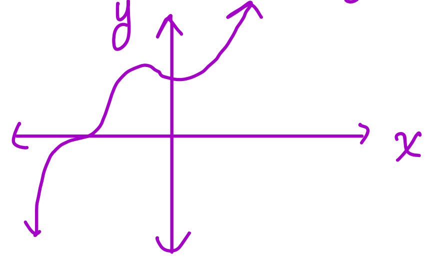
2. It is possible for a cubic polynomial with real coefficients to have all non-real zeros? To have exactly 2 non-real zeros?

No b/c if the coefficients are real non-real zeros have to come in conjugate pairs.

→ yes

3. Make a sketch of a 5^{th} degree polynomial which has 1 real zero of multiplicity 3 and 2 non-real zeros.

graph can vary



to show non-real zero
rel min above the x-axis
rel max below the x-axis
terrace pt above or below x-axis

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PCH: More Practice with Vertical, Horizontal and Oblique Asymptotes — Ms. Loughran

RF

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote Does graph intersect HA?	Oblique Asymptote Does graph intersect the OA?	*x*-intercept(s)	*y*-intercept
① $y=\frac{x^2+x-2}{x^2-x-6}$ $\frac{(x+2)(x-1)}{(x-3)(x+2)}$; $y=\frac{x-1}{x-3}$	$(-2, \frac{3}{5})$	$x-3=0$, $x=3$	$y=1$; $\frac{x-1}{x-3}=1$, $x-1=x-3$, $-1\neq-3$ No	none	$\frac{x-1}{x-3}=0$, $x-1=0$, $x=1$, $(1,0)$	$y=\frac{0-1}{0-3}=\frac{1}{3}$, $(0,\frac{1}{3})$
② $y=\frac{3}{x-2}$	none	$x=2$	$y=0$; $\frac{3}{x-2}=0$, $3\neq0$ no	none	none	$y=\frac{3}{0-2}$, $(0,-\frac{3}{2})$
③ $y=\frac{2x^2}{x^2-1}$	none	$x=\pm1$	$y=2$; $\frac{2x^2}{x^2-1}=2$, $2x^2=2x^2-2$, $0\neq-2$ no	none	$\frac{2x^2}{x^2-1}=0$, $2x^2=0$, $x^2=0$, $x=0$, $(0,0)$	$(0,0)$
④ $y=\frac{2x-1}{x}$	none	$x=0$	$y=2$; $\frac{2x-1}{x}=2$, $2x-1=2x$, $-1\neq0$ No	none	$\frac{2x-1}{x}=0$, $2x-1=0$, $x=\frac{1}{2}$, $(\frac{1}{2},0)$	$y=\frac{2(0)-1}{0}$ none
⑤ $y=\frac{x^2+x-12}{x^2-9}$ $\frac{(x+4)(x-3)}{(x+3)(x-3)}$; $y=\frac{x+4}{x+3}$	$(3,\frac{7}{6})$	$x=-3$	$y=1$; $\frac{x+4}{x+3}=1$, $x+4\neq x+3$ no	none	$\frac{x+4}{x+3}=0$, $x+4=0$, $x=-4$, $(-4,0)$	$y=\frac{0+4}{0+3}=\frac{4}{3}$, $(0,\frac{4}{3})$
⑥ $y=\frac{x^2-4}{x+3}$	none	$x=-3$	none	$y=x-3$; no	$\frac{x^2-4}{x+3}=0$, $x^2-4=0$, $x=\pm2$, $(\pm2,0)$	$y=\frac{0^2-4}{0+3}=\frac{-4}{3}$, $(0,-\frac{4}{3})$

$$\begin{array}{r|rrr} -3 & 1 & 0 & -4 \\ & & -3 & 9 \\ \hline & 1 & -3 & 5 \end{array}$$

$\frac{x^2-4}{x+3}=x-3$

$x^2-4\neq x^2-9$

Function	Hole(s)	Vertical Asymptote (s)	Horizontal Asymptote Does graph intersect the HA?	Oblique Asymptote Does graph intersect the OA?	*x*-intercept(s)	*y*-intercept
⑦ $x(x-1)$ $y=\dfrac{x^2-x}{x+1}$	none	$x=-1$	none	$-1 \vert\ 1\ \ -1\ \ 0$ $-1\ \ 2$ $1\ \ -2\ \ 2$ $y=x-2$ $\frac{x^2-x}{x+1}=x-2$ $x^2-x=x^2-x-2$ $0\neq-2$ no	$\frac{x(x-1)}{x+1}=0$ $x(x-1)=0$ $x=0 \quad x=1$ $(0,0)$ $(1,0)$	$(0,0)$
⑧ $(x-2)(x+1)$ $y=\dfrac{x^2-x-2}{x-1}$	none	$x=1$	none	$1 \vert\ 1\ \ -1\ \ -2$ $1\ \ 0$ $1\ \ 0\ \ -2$ $y=x$ $\frac{x^2-x-2}{x-1}=x$ $x^2-x-2=x^2-x$ $-2\neq0$ no	$\frac{(x-2)(x+1)}{x-1}=0$ $(x-2)(x+1)=0$ $x=2 \mid x=-1$ $(2,0),\ (-1,0)$	$y=\frac{-2}{-1}=2$ $(0,2)$
⑨ $y=\dfrac{x+1}{x^2+3x+2}=\dfrac{1}{x+2}$ $(x+2)(x+1)$	$(-1,1)$	$x=-2$	$y=0$ $\frac{1}{x+2}=0$ $1\neq0$ no	none	none (arrow to HA column)	$y=\frac{1}{0+2}=\frac{1}{2}$ $(0,\frac{1}{2})$
⑩ $(x-3)(x+3)$ $y=\dfrac{x^2-9}{x^2-2x-3}=\dfrac{x+3}{x+1}$ $(x-3)(x+1)$	$(3,\frac{3}{2})$	$x=-1$	$y=1$ $\frac{x+3}{x+1}=1$ $x+3=x+1$ $3\neq1$ no	none	$0=\frac{x+3}{x+1}$ $0=x+3$ $x=-3$ $(-3,0)$	$y=\frac{3}{1}=3$ $(0,3)$
⑪ $y=\dfrac{2x^3-17x^2-8x-9}{3-x^2}$	← Did this one already					
⑫ $3x^2(x-3)+1(x-3)$ $y=\dfrac{3x^3-9x^2+x-3}{x-3}$ $y=\frac{(3x^2+1)(x-3)}{x-3}=3x^2+1$	$(3,28)$	none	none	none	$0=3x^2+1$ $-1=3x^2$ imaginary none	$y=3(0)^2+1=1$ $(0,1)$

Homework 12-12

Name:____ Date:__________

PCH: Practice with Vertical, Horizontal and Oblique Asymptotes — Ms. Loughran

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote Does graph intersect HA?	Oblique Asymptote Does graph intersect the OA?	*x*-intercept(s)	*y*-intercept
① $y=\frac{x-5}{x^2-4x-5}$ $(x-5)(x+1)$ $=\frac{1}{x+1}$	$(5, \frac{1}{6})$	$x+1=0$ $x=-1$	$y=0$ $\frac{1}{x+1}=0$ $1\neq 0$ No	none	$\frac{1}{x+1}=0$ none	$y=\frac{1}{0+1}=1$ $(0,1)$
② $y=\frac{2x+1}{x^2}$	none	$x=0$	$y=0$ $\frac{2x+1}{x^2}=0$ $2x+1=0$ $x=-\frac{1}{2}$ $(-\frac{1}{2},0)$	none	$\frac{2x+1}{x^2}=0$ $0=2x+1$ $-\frac{1}{2}=x$ $(-\frac{1}{2},0)$	$y=\frac{2(0)+1}{0^2}$ none
③ $y=\frac{x-5}{x^2+1}$	none	none	$y=0$ $\frac{x-5}{x^2+1}=0$ $x-5=0$ $x=5$ $(5,0)$	none	$\frac{x-5}{x^2+1}=0$ $x-5=0$ $x=5$ $(5,0)$	$y=\frac{0-5}{0^2+1}=-5$ $(0,-5)$
④ $y=\frac{2x}{x^2-x-6}$	none	$x^2-x-6=0$ $(x-3)(x+2)=0$ $x=3,-2$	$y=0$ $\frac{2x}{x^2-x-6}=0$ $2x=0$ $x=0$ $(0,0)$	none	$(0,0)$	$(0,0)$
⑤ $y=\frac{-3x^2+2}{x-1}$	none	$x-1=0$ $x=1$	none	$1 \mid -3\ \ 0\ \ 2$ $-3\ \ -3$ $-3\ \ -3\ \ -1$ $y=-3x-3$	$\frac{-3x^2+2}{x-1}=0$ $-3x^2+2=0$ $2=3x^2$ $\frac{2}{3}=x^2$ $(\pm\sqrt{\frac{2}{3}},0)$	$y=\frac{-3(0)^2+2}{0-1}=\frac{2}{-1}$ $(0,-2)$
⑥ $(2x^2+x+1)(x-9)$ $y=\frac{2x^3-17x^2-8x-9}{3-x^2}$	none	$3-x^2=0$ $3=x^2$ $\pm\sqrt{3}=x$	none	$y=-2x+17$ yes $(-30,77)$	$(2x^2+x+1)(x-9)=0$ $x=\frac{-1\pm\sqrt{1^2-4(2)(1)}}{2(1)}$ imaginary $x=9$ $(9,0)$	$y=\frac{-9}{3}=-3$ $(0,-3)$

$$\frac{-3x^2+2}{x-1}=-3x-3$$

$$\begin{array}{r} -2x+17 \\ -x^2+3\ \overline{)\ 2x^3-17x^2-8x-9} \\ 2x^3\qquad\quad -6x \\ \hline -17x^2-2x-9 \end{array}$$

$\frac{2x^3-17x^2-8x-9}{3-x^2}=-2x+17$

$2x^3-17x^2-8x-9=(3-x^2)(-2x+17)$

$2x^3-17x^2-8x-9=-6x+2x^3+51-17x^2$

$2x^3-17x^2-8x-9=2x^3-17x^2-6x+51$

$-8x-9=-6x+51$

$-2x=60$

$x=-30$

Classwork/Homework 12-13

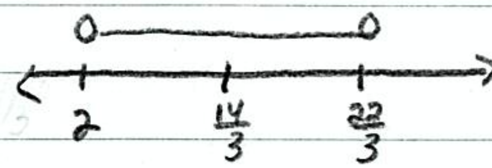
Some Review Problems

① $3\left|7-\frac{3}{2}x\right|<4$

$\left|\frac{3}{2}x-7\right|<4$

$\frac{2}{3}\cdot\frac{3}{2}\left|x-\frac{14}{3}\right|<4\cdot\frac{2}{3}$

$\left|x-\frac{14}{3}\right|<\frac{8}{3}$

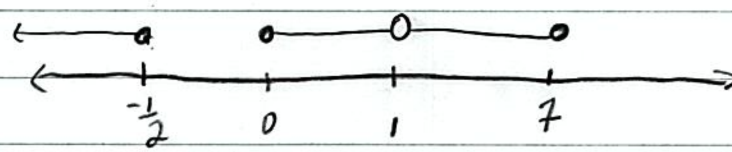


x's distance from $\frac{14}{3}<\frac{8}{3}$

$\left(2,\frac{22}{3}\right)$

② $\frac{x(7-x)(2x+1)}{(x-1)^2}\geq 0$

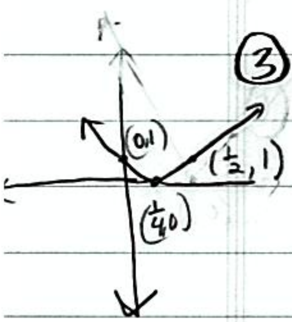
$(x-1)^2$ always +



x	−	−	+	+	+
$(7-x)$	+	+	+	+	−
$(2x+1)$	−	+	+	+	+
numerator	+	−	+	+	−

$\left(-\infty,-\frac{1}{2}\right]\cup[0,1)\cup(1,7]$

③ $y=|4x-1|$



$4x-1$ if $4x-1\geq 0$ $\quad 4x\geq 1$ $\quad x\geq\frac{1}{4}$

$-4x+1$ if $4x-1<0$ $\quad 4x<1$ $\quad x<\frac{1}{4}$

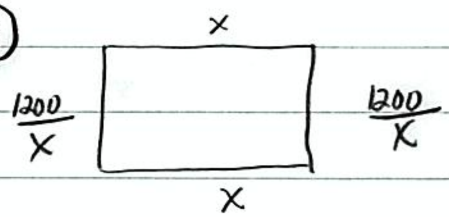
(4) $f(3) = -2$ $\quad (3, -2)$ $\quad \frac{1-(-2)}{-7-3} = \frac{3}{-10}$

$f(-7) = 1$ $\quad (-7, 1)$

$y - 1 = -3/10(x+7)$

$f(x) - 1 = -3/10\,(x+7)$ OR $f(x) + 2 = -3/10(x-3)$

(5)



$x > 0$

$A = lw$

$1200 = x \cdot w$

$\frac{1200}{x} = w$

$\frac{2400}{x} + x$

$C(x) = 5x + 3\left(\frac{1200}{x} + \frac{1200}{x} + x\right)$

$C(x) = 5x + \frac{7200}{x} + 3x$

$C(x) = 8x + 7200/x \quad , x > 0$

(6) $(3a+1)^2 - 6(3a+1) + 8$

$x^2 - 6x + 8$

$(x-4)(x-2)$

$(3a+1-4)(3a+1-2)$

$(3a-3)(3a-1)$

$3(a-1)(3a-1)$

(b) $1000x^3y^9 - 125a^{12}b^{15}$

$a = 2xy^3$
$b = a^4b^5$

$125(8x^3y^9 - a^{12}b^{15})$

$125(2xy^3 - a^4b^5)(4x^2y^6 + 2xy^3a^4b^5 + a^8b^{10})$

(c) $x^4 - 15x^2 + 9 + 9x^2 - 9x^2$

$x^4 - 6x^2 + 9 - 9x^2$

$(x^2 - 3)^2 - 9x^2$

$(x^2 - 3x - 3)(x^2 + 3x - 3)$

(7) $\frac{4y^2-9}{2y^2-9y-18} \div \frac{2y^2+y-3}{y^2+5y-6}$

$\frac{(2y-3)\cancel{(2y+3)}}{(2y+3)(y-6)} \cdot \frac{(y+6)\cancel{(y-1)}}{\cancel{(2y+3)}\cancel{(y-1)}}$

$\frac{(2y-3)(y+6)}{(2y+3)(y-6)}$ $y \neq 1, \pm 3/2, \pm 6$

(8) $y = -\frac{3}{2x-3} + 5$

$x = \frac{-3}{2y-3} + 5$

$x-5 = \frac{-3}{2y-3}$

$(x-5)(2y-3) = -3$

$2y-3 = \frac{-3}{x-5}$

$\frac{1}{2} \cdot 2y = \left(\frac{-3}{x-5} + 3\right) \cdot \frac{1}{2}$

$y = \frac{-3}{2(x-5)} + \frac{3}{2}$

⑨ $f(x)=\sqrt{x}$ $\qquad$ $g(x)=\sqrt{9-x}$

$9-x\geq 0$
$-x\geq -9$
$x\leq 9$

(a) $d_f: x\geq 0$ $\qquad$ $d_g: x\leq 9$

$(f\circ g)(x)=\sqrt{\sqrt{9-x}} = \sqrt[4]{9-x}$ $\qquad$ $d: x\leq 9$ $\qquad$ $d_{f\circ g}: x\leq 9$

$(g\circ f)(x)=\sqrt{9-\sqrt{x}}$ $\qquad$ $d_{g\circ f}: 0\leq x\leq 81$

$9-\sqrt{x}\geq 0$
$-\sqrt{x}\geq -9$
$\sqrt{x}\leq 9$
$x\leq 81$

$(f\circ f)(x)=\sqrt{\sqrt{x}} = \sqrt[4]{x}$ $\qquad$ $d_{f\circ f}: x\geq 0$

$(g\circ g)(x)=\sqrt{9-\sqrt{9-x}}$ $\qquad$ $d_{g\circ g}: x\geq 72$

$9-\sqrt{9-x}\geq 0$
$-\sqrt{9-x}\geq -9$
$\sqrt{9-x}\leq 9$
$9-x\leq 81$
$-x\leq 72$
$x\geq 72$

(9)

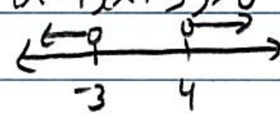
(b) $f(x) = \frac{1}{\sqrt{x}}$ $\qquad$ $g(x) = x^2 - x - 12$

$d_g : \mathbb{R}$

$d_f : x > 0$

$(f \circ g)(x) = \frac{1}{\sqrt{x^2 - x - 12}}$ $\qquad$ $x^2 - x - 12 > 0$ $\qquad$ $d_{f\circ g} : x < -3 \vee x > 4$

$(x-4)(x+3) > 0$



$(g \circ f)(x) = (\sqrt{x})^2 - \sqrt{x} - 12$ $\qquad$ $d_{g\circ f} : x > 0$

$x - \sqrt{x} - 12$

$x \geq 0$

$(f \circ f)(x) = \frac{1}{\sqrt{\frac{1}{\sqrt{x}}}} = \frac{1}{\frac{1}{\sqrt[4]{x}}} = \sqrt[4]{x}$ $\qquad$ $d_{f\circ f} : x > 0$

$(g \circ g)(x) = (x^2 - x - 12)^2 - (x^2 - x - 12) - 12$ $\qquad$ $d_{g\circ g} : \mathbb{R}$