Name:____ Date:___________

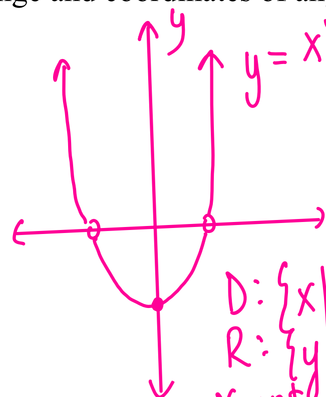
PCH: Vertical and Horizontal Asymptotes Ms. Loughran

Do Now:

1. Graph $y = \dfrac{x^4 - 2x^2 + 1}{x^2 - 1}$. State the domain, range and coordinates of any hole(s), x- and y-intercepts.

$$y = \frac{\cancel{(x^2-1)}(x^2-1)}{\cancel{x^2-1}} = x^2 - 1$$

x^2 ↓1



$y = \dfrac{x^4 - 2x^2 + 1}{x^2 - 1}$

holes: $(1, 0)$, $(-1, 0)$

D: $\{x \mid x \neq \pm 1\}$

R: $\{y \mid -1 \leq y < 0 \vee y > 0\}$

x-int: none

y-int: $(0, -1)$

A vertical asymptote is a vertical line that guides the graph of the function but is not part of it. It can never be crossed by the graph because it occurs at the x-value that is not in the domain of the function

A horizontal asymptote describes a function's "end behavior." That means how the graph behaves as x approaches $\pm\infty$.

Higher powered exponents really drive the end behavior of the graph.

Examples:

1. What is the end behavior (EB) of $y = \dfrac{x^3 + 5}{2x^3 + x^2 + 1}$?

$$y = \frac{\frac{x^3}{x^3} + \frac{5}{x^3}}{\frac{2x^3}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}} = \frac{1 + \frac{5}{x^3}}{2 + \frac{1}{x} + \frac{1}{x^3}}$$

as $x \to \pm\infty$

$$y = \frac{1 + 0}{2 + 0 + 0} = \frac{1}{2}$$

EB: $y = \frac{1}{2}$ (horizontal)

2. What is the end behavior of $y = \dfrac{2x-3}{x^2+2}$?

$$y = \dfrac{\dfrac{2x}{x^2} - \dfrac{3}{x^2}}{\dfrac{x^2}{x^2} + \dfrac{2}{x^2}}$$

$$y = \dfrac{\dfrac{2}{x} - \dfrac{3}{x^2}}{1 + \dfrac{2}{x^2}}$$

as $x \to \pm\infty$

$$y = \dfrac{0-0}{1+0} = \dfrac{0}{1} \quad 0$$

EB: $y=0$ (horizontal)

3. What is the end behavior of $y = \dfrac{x^2-4}{x^2-2x+3}$?

$$y = \dfrac{\dfrac{x^2}{x^2} - \dfrac{4}{x^2}}{\dfrac{x^2}{x^2} - \dfrac{2x}{x^2} + \dfrac{3}{x^2}}$$

$$y = \dfrac{1 - \dfrac{4}{x^2}}{1 - \dfrac{2}{x} + \dfrac{3}{x^2}}$$

as $x \to \pm\infty$ $y = \dfrac{1-0}{1-0+0} = \dfrac{1}{1} = 1$

EB: $y=1$ (horizontal)

4. What is the end behavior of $y = \dfrac{x^2}{x+1}$?

$$y = \dfrac{\dfrac{x^2}{x^2}}{\dfrac{x}{x^2} + \dfrac{1}{x^2}}$$

as $x \to \pm\infty$

$$y = \dfrac{1}{\dfrac{1}{x} + \dfrac{1}{x^2}}_{0} = \dfrac{1}{\dfrac{1}{x}} = x$$

EB: $y=x$ (oblique)

no horizontal asymptote

Let r be the **REDUCED** rational function

$$r(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0}$$

1. The vertical asymptotes of r are the lines $x = a$, where a is a zero of the denominator.

   In other words: to find vertical asymptotes, we set the denominator of the REDUCED function equal to 0 and solve.

2. (a) If $n < m$, then r has a horizontal asymptote of $y = 0$

   In other words: if the degree of the numerator is less than the degree of the denominator, then the horizontal asymptote is $y = 0$.

   (b) If $n = m$, then r has a horizontal asymptote of $y =$ ratio of the leading coefficients

   In other words: if the degree of the numerator = the degree of the denominator, then the horizontal asymptote is $y =$ ratio of the leading coefficients.

   (c) If $n > m$, then r has. no horizontal asymptote

   In other words: if the degree of the numerator > the degree of the denominator, then there is no horizontal asymptote.

Graphs can intersect horizontal asymptotes, but can never intersect a vertical asymptote. So you must always check if a graph intersects its horizontal asymptote.

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote Does graph intersect HA?	x-intercept(s)	y-intercept
1 $y=\frac{1-x}{x+3}$	none	$x+3=0$ $x=-3$	$y=\frac{-1}{1}=-1$ $\frac{1-x}{x+3}=-1$ NO $1-x=-x-3$ $1\neq-3$	$\frac{1-x}{x+3}=0$ $1-x=0$ $x=1$ $(1,0)$	$y=\frac{1-0}{0+3}=\frac{1}{3}$ $(0,\frac{1}{3})$
RF $y=\frac{1}{x+2}$ 2 $y=\frac{\cancel{x-2}}{x^2-4}$ $(x+2)\cancel{(x-2)}$	$(2,\frac{1}{4})$	$x+2=0$ $x=-2$	$y=0$ $\frac{1}{x+2}=0$ $1\neq0$ NO	$\frac{1}{x+2}=0$ $1\neq0$ none	$y=\frac{1}{0+2}$ $(0,\frac{1}{2})$
$y=x-5$ 3 $(x-5)\cancel{(x+4)}$ $y=\frac{x^2-x-20}{\cancel{x+4}}$	$(-4,-9)$	none	none	$y=x-5$ $0=x-5$ $5=x$ $(5,0)$	$y=0-5$ $y=-5$ $(0,-5)$
4 $(x-5)(x+4)$ $y=\frac{x^2-x-20}{x+1}$	none	$x+1=0$ $x=-1$	none	$0=\frac{(x-5)(x+4)}{x+1}$ $0=(x-5)(x+4)$ $(5,0),(-4,0)$	$y=\frac{0-0-20}{0+1}=-20$ $(0,-20)$
5 $y=\frac{2x^3}{x^3+x}$					
6 $y=\frac{x-1}{x^2-4}$					

Homework 12-08

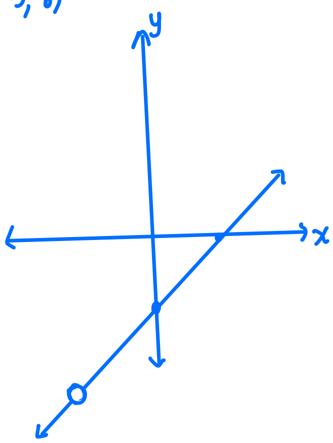
Sketch the graph of each of the following. State the domain, range, and any intercepts.

1. $y=\dfrac{x^2-9}{x+3}$

$y=\dfrac{(x-3)\cancel{(x+3)}}{\cancel{x+3}}=x-3$

hole: $(-3,-6)$

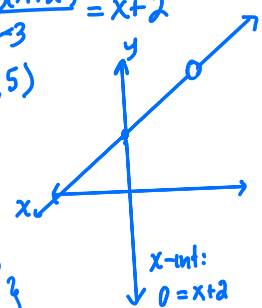
D: $\{x \mid x\neq -3\}$
R: $\{y \mid y\neq -6\}$
x-int: $(3,0)$
y-int: $(0,-3)$



2. $y=\dfrac{x^2-x-6}{x-3}$

$y=\dfrac{\cancel{(x-3)}(x+2)}{\cancel{x-3}}=x+2$

hole: $(3,5)$



x-int:
$0=x+2$
$-2=x$

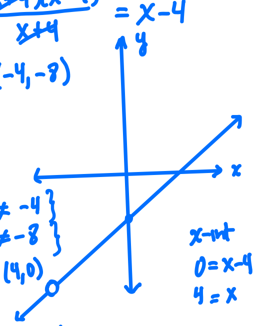
D: $\{x \mid x\neq 3\}$
R: $\{y \mid y\neq 5\}$
x-int: $(-2,0)$
y-int: $(0,2)$

3. $y=\dfrac{x^2-16}{x+4}$

$y=\dfrac{\cancel{(x+4)}(x-4)}{\cancel{x+4}}=x-4$

hole: $(-4,-8)$

D: $\{x \mid x\neq -4\}$
R: $\{y \mid y\neq -8\}$
x-int: $(4,0)$



x-int
$0=x-4$
$4=x$

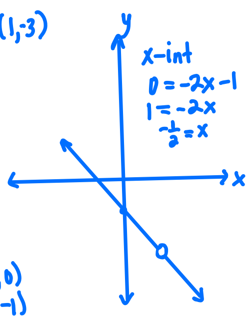
y-int: $(0,-4)$

4. $y=\dfrac{1+x-2x^2}{x-1}$ ($-2x^2+x+1$)

$y=\dfrac{-(2x^2-x-1)}{(x-1)}$

$y=-\dfrac{(2x+1)\cancel{(x-1)}}{\cancel{x-1}}=-2x-1$

hole: $(1,-3)$



x-int
$0=-2x-1$
$1=-2x$
$-\frac{1}{2}=x$

D: $\{x \mid x\neq 1\}$
R: $\{y \mid y\neq -3\}$
x-int: $(-\frac{1}{2},0)$
y-int: $(0,-1)$

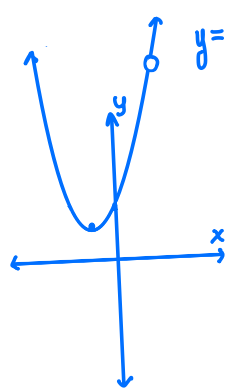
5. $y=\dfrac{x^3-8}{x-2}$

$y=\dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$y=x^2+2x+4$

hole: $(2,12)$

$y=x^2+2x+4$
$y=x^2+2x+1-1+4$
$y=(x+1)^2+3$
vertex: $(-1,3)$



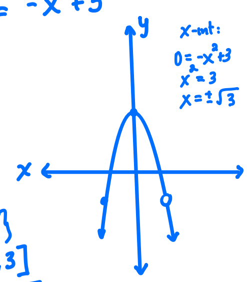
D: $\{x \mid x\neq 2\}$
R: $[3,\infty)$
x-int: none
y-int: $(0,4)$

6. $y=\dfrac{x^3-2x^2-3x+6}{2-x}$ ($x^2(x-2)-3(x-2)$)

$y=\dfrac{(x^2-3)\cancel{(x-2)}}{\cancel{2-x}}=-x^2+3$

hole: $(2,-1)$

x-int:
$0=-x^2+3$
$x^2=3$
$x=\pm\sqrt{3}$



D: $\{x \mid x\neq 2\}$
R: $(-\infty,3]$
x-int: $(\pm\sqrt{3},0)$
y-int: $(0,3)$

49–52 ■ Graph the functions on the same screen using the given viewing rectangle. How is each graph related to the graph in part (a)?

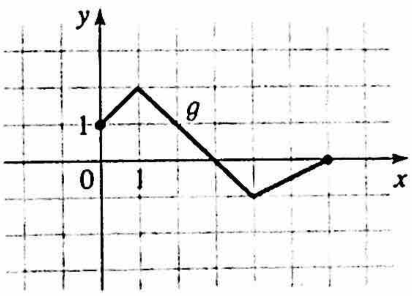
49. Viewing rectangle $[-8, 8]$ by $[-2, 8]$
(a) $y = \sqrt[4]{x}$ (b) $y = \sqrt[4]{x + 5}$
(c) $y = 2\sqrt[4]{x + 5}$ (d) $y = 4 + 2\sqrt[4]{x + 5}$

50. Viewing rectangle $[-8, 8]$ by $[-6, 6]$
(a) $y = |x|$ (b) $y = -|x|$
(c) $y = -3|x|$ (d) $y = -3|x - 5|$

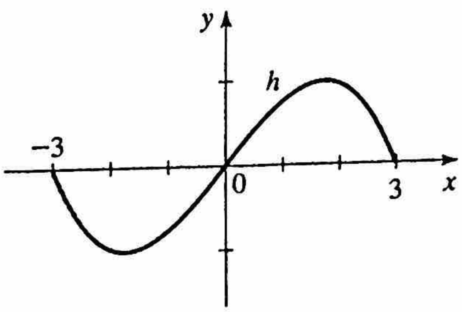
51. Viewing rectangle $[-4, 6]$ by $[-4, 4]$
(a) $y = x^6$ (b) $y = \frac{1}{3}x^6$
(c) $y = -\frac{1}{3}x^6$ (d) $y = -\frac{1}{3}(x - 4)^6$

52. Viewing rectangle $[-6, 6]$ by $[-4, 4]$
(a) $y = \dfrac{1}{\sqrt{x}}$ (b) $y = \dfrac{1}{\sqrt{x + 3}}$
(c) $y = \dfrac{1}{2\sqrt{x + 3}}$ (d) $y = \dfrac{1}{2\sqrt{x + 3}} - 3$

53. The graph of g is given. Use it to graph each of the following functions.
(a) $y = g(2x)$ (b) $y = g(\frac{1}{2}x)$

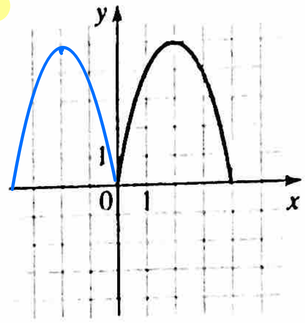


54. The graph of h is given. Use it to graph each of the following functions.
(a) $y = h(3x)$ (b) $y = h(\frac{1}{3}x)$

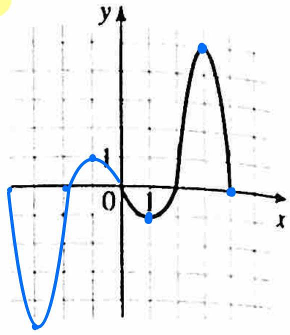


55–56 ■ The graph of a function defined for $x \geq 0$ is given. Complete the graph for $x < 0$ to make
(a) an even function
(b) an odd function

55.



56.



57–58 ■ Use the graph of $f(x) = [\![x]\!]$ described on pages 162–163 to graph the indicated function.

57. $y = [\![2x]\!]$ **58.** $y = [\![\frac{1}{4}x]\!]$

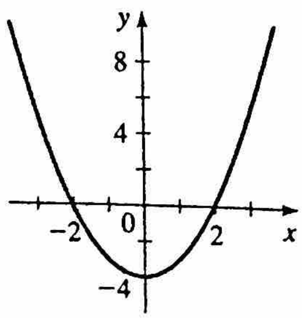
59. If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle $[-5, 5]$ by $[-4, 4]$. How is each graph related to the graph in part (a)?
(a) $y = f(x)$ (b) $y = f(2x)$ (c) $y = f(\frac{1}{2}x)$

60. If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle $[-5, 5]$ by $[-4, 4]$. How is each graph related to the graph in part (a)?
(a) $y = f(x)$ (b) $y = f(-x)$ (c) $y = -f(-x)$
(d) $y = f(-2x)$ (e) $y = f(-\frac{1}{2}x)$

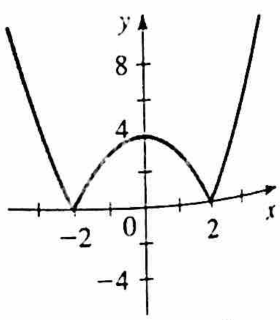
61–68 ■ Determine whether the function f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

61. $f(x) = x^{-2}$ even
62. $f(x) = x^{-3}$ odd
63. $f(x) = x^2 + x$ neither
64. $f(x) = x^4 - 4x^2$ even
65. $f(x) = x^3 - x$ odd
66. $f(x) = 3x^3 + 2x^2 + 1$ neither
67. $f(x) = 1 - \sqrt[3]{x}$ neither
68. $f(x) = x + \dfrac{1}{x}$ odd

69. The graphs of $f(x) = x^2 - 4$ and $g(x) = |x^2 - 4|$ are shown. Explain how the graph of g is obtained from the graph of f.



$f(x) = x^2 - 4$



$g(x) = |x^2 - 4|$