Name:_________________________________ Date:________
PCH: Sketching Polynomials without a Graphing Calculator Ms. Loughran

Do Now:

Sketch the general graph of each function without your graphing calculator. Your sketch should contain both the *x*- and *y*-intercepts and indicate the end behavior of the graph.

1. $f(x) = -(x + 3)(x + 2)(x - 1)$

2. $f(x) = (x + 5)^2(x + 3)$

3. $f(x) = x^3 + 2x^2 - 36x - 72$

POLYNOMIAL GRAPH SUMMARY

The following are general statements about the graph of a polynomial function $y = P(x)$.

1. A polynomial function is continuous — i.e. it is defined for all real values of x and can be drawn without lifting the pencil from the paper. Therefore, it has no holes or vertical asymptotes, nor does it have any "jumps" or other gaps.

2. The "turns" taken by the graph are smooth. There are no sharp corners or points.

3. As $|x|$ gets very large, the points of the graph move further and further away from the x-axis. And, there are no horizontal or oblique asymptotes.

The following are statements about the roots of the polynomial equation $P(x) = 0$ with real coefficients and the curve $y = P(x)$.

1. A polynomial equation of degree n has n roots, real and/or nonreal, counting multiplicity.

2. An intersection (not tangency or terrace point) of the curve and the x-axis indicates one real root.

3. A point of tangency located on the x-axis indicates a real root of even multiplicity (2 equal roots, 4 equal roots, etc.).

4. A terrace point located on the x-axis indicates a real root of odd multiplicity (3 equal roots, 5 equal roots, etc.).

5. Non-real roots occur in conjugate pairs.

6. A relative maximum below the x-axis, or a relative minimum above the x-axis, or a terrace point above or below the x-axis indicates one pair of conjugate nonreal roots or a multiplicity of conjugate nonreal roots.
NOTE: The converse of this statement is false (consider $y = x^4 - 1$).

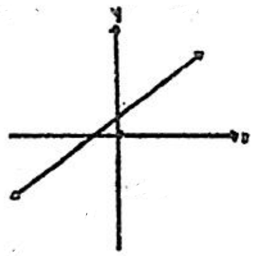
Graphs of Polynomial Functions: End behavior

	Odd degree		Even degree	
Sign of Leading Coefficient	Positive	Negative	Positive	Negative
End behavior	↙ ↗	↖ ↘	↖ ↗	↙ ↘

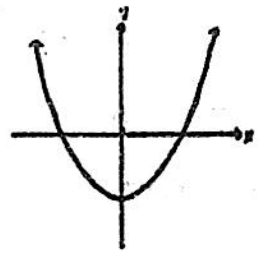
Pre-Calculus

Each graph is a polynomial of degree n. Indicate the number of real roots and the number of non-real roots of P(x) = 0. Describe the multiplicity of roots when the multiplicity is greater than 1.

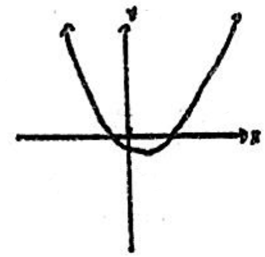
1. $n = 1$



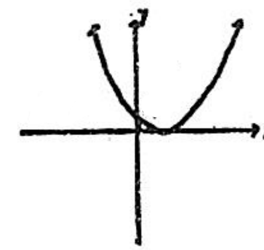
2. $n = 2$



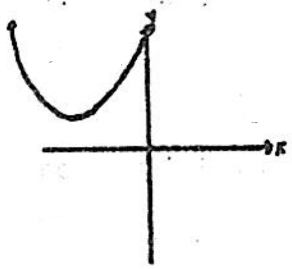
3. $n = 2$



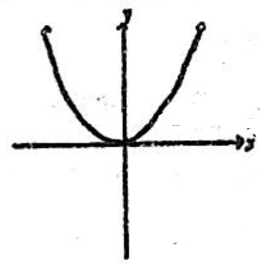
4. $n = 2$



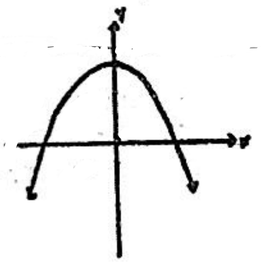
5. $n = 2$



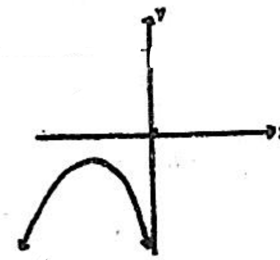
6. $n = 2$



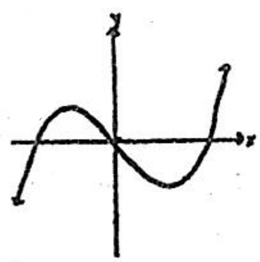
7. $n = 2$



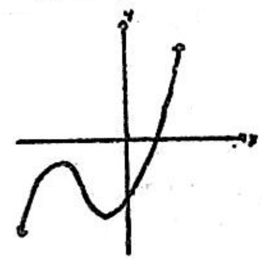
8. $n = 2$



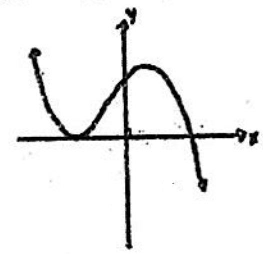
9. $n = 3$



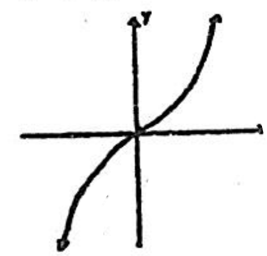
10. $n = 3$



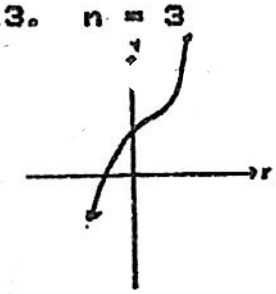
11. $n = 3$



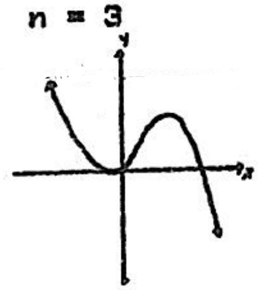
12. $n = 3$



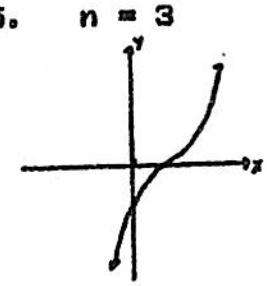
13. $n = 3$



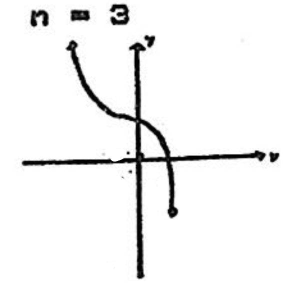
14. $n = 3$



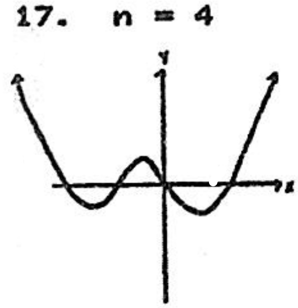
15. $n = 3$



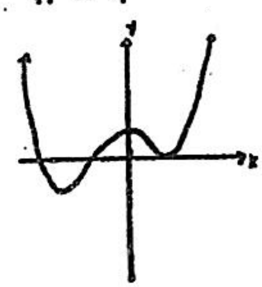
16. $n = 3$



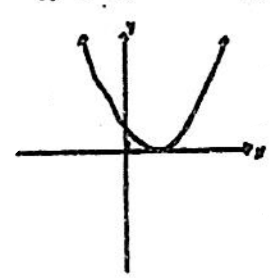
17. $n = 4$



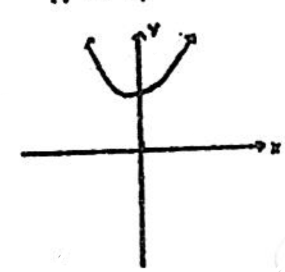
18. $n = 4$



19. $n = 4$



20. $n = 4$



SKETCHING POLYNOMIAL GRAPHS

Sketch each of the following:

1. $y = x^2$
2. $y = (x - 2)^2$
3. $y = -(x + 3)^2$
4. $y = (x + 1)(x - 2)(x - 3)$
5. $y = (x + 1)(x - 2)(3 - x)$
6. $y = x(x - 1)^2$
7. $y = -x(x - 2)^2$
8. $y = x^2(x + 2)(x - 2)$
9. $y = x^3(x - 3)$
10. $y = -x(x - 1)(x + 2)$
11. $y = x^3 - 6x^2 + 9x$
12. $y = x^2(x - 4)$
13. $y = (x + 2)^3$
14. $y = x(x - 1)(x - 3)(x + 2)$
15. $y = x(1 - x)(x - 2)(x + 3)$
16. $y = x(x + 1)(x - 3)^2$
17. $y = x^4 - 5x^2 + 4$
18. $y = 3x^3 - x^4$
19. $y = x^2(x - 1)(x + 2)(x + 3)$
20. $y = (1 - x)(2 - x)(3 - x)(4 - x)(5 - x)$

21. Find a cubic polynomial whose graph intersects the x-axis at 3 and is tangent to the x-axis at -1. Is this answer unique?

22. Find the cubic polynomial whose graph passes through the points (2,0) and (4,6) and is tangent to the x-axis at the origin.

23. Find the cubic polynomial whose y-intercept is 9 and whose x-intercepts are 1, 2, and -3.

24. Find the 4th degree polynomial whose graph passes through (0,6) and is tangent to the x-axis at (3,0) and $(-2,0)$.