Name: _____ Date: _____
PCH: General Solutions to Trig Equations Ms. Loughran

Do Now:

Find all solutions of the equation in the interval $(0, 2\pi]$.

1. $\sin x = 1$

2. $\cos^2 x = \frac{3}{4}$

3. $\sec x - 2 = 0$

Now let's write what is called the general solutions for questions 1-3.

Classwork:

Find all solutions of each equation.

1. $\tan 2x = \frac{1}{\sqrt{3}}$

2. $\cos \frac{x}{2} + 1 = 0$

3. $2\sin^2 x - 1 = 0$

4. $4\cos^2 x - 1 = 0$

5. $\sin^2 x = 2\sin x + 3$

6. $\sin^2 x - \cos^2 x = 0$

From your textbook:

## 7.5 Exercises

**1–40** ■ Find all solutions of the equation.

**1.** $\cos x + 1 = 0$

**2.** $\sin x + 1 = 0$

**3.** $2 \sin x - 1 = 0$

**4.** $\sqrt{2} \cos x - 1 = 0$

**5.** $\sqrt{3} \tan x + 1 = 0$

**6.** $\cot x + 1 = 0$

**7.** $4 \cos^2 x - 1 = 0$

**8.** $2 \cos^2 x - 1 = 0$

**9.** $\sec^2 x - 2 = 0$

**10.** $\csc^2 x - 4 = 0$

**11.** $3 \csc^2 x - 4 = 0$

**12.** $1 - \tan^2 x = 0$

**13.** $\cos x (2 \sin x + 1) = 0$

**14.** $\sec x (2 \cos x - \sqrt{2}) = 0$

**15.** $(\tan x + \sqrt{3})(\cos x + 2) = 0$

**16.** $(2 \cos x + \sqrt{3})(2 \sin x - 1) = 0$

**17.** $\cos x \sin x - 2 \cos x = 0$

**18.** $\tan x \sin x + \sin x = 0$

**19.** $4 \cos^2 x - 4 \cos x + 1 = 0$

**20.** $2 \sin^2 x - \sin x - 1 = 0$

**21.** $\sin^2 x = 2 \sin x + 3$

**22.** $3 \tan^3 x = \tan x$

**23.** $\sin^2 x = 4 - 2 \cos^2 x$

**24.** $2 \cos^2 x + \sin x = 1$

**25.** $2 \sin 3x + 1 = 0$

**26.** $2 \cos 2x + 1 = 0$

**27.** $\sec 4x - 2 = 0$

**28.** $\sqrt{3} \tan 3x + 1 = 0$

**29.** $\sqrt{3} \sin 2x = \cos 2x$

**30.** $\cos 3x = \sin 3x$

**31.** $\cos \dfrac{x}{2} - 1 = 0$

**32.** $2 \sin \dfrac{x}{3} + \sqrt{3} = 0$

**33.** $\tan \dfrac{x}{4} + \sqrt{3} = 0$

**34.** $\sec \dfrac{x}{2} = \cos \dfrac{x}{2}$

**35.** $\tan^5 x - 9 \tan x = 0$

**36.** $3 \tan^3 x - 3 \tan^2 x - \tan x + 1 = 0$

**37.** $4 \sin x \cos x + 2 \sin x - 2 \cos x - 1 = 0$

**38.** $\sin 2x = 2 \tan 2x$

**39.** $\cos^2 2x - \sin^2 2x = 0$

**40.** $\sec x - \tan x = \cos x$

**41–48** ■ Find all solutions of the equation in the interval $[0, 2\pi)$.

**41.** $2 \cos 3x = 1$

**42.** $3 \csc^2 x = 4$

**43.** $2 \sin x \tan x - \tan x = 1 - 2 \sin x$

**44.** $\sec x \tan x - \cos x \cot x = \sin x$

**45.** $\tan x - 3 \cot x = 0$

**46.** $2 \sin^2 x - \cos x = 1$

**47.** $\tan 3x + 1 = \sec 3x$

**48.** $3 \sec^2 x + 4 \cos^2 x = 7$

**49–56** ■ **(a)** Find all solutions of the equation. **(b)** Use a calculator to solve the equation in the interval $[0, 2\pi)$, correct to five decimal places.

**49.** $\cos x = 0.4$

**50.** $2 \tan x = 13$

**51.** $\sec x - 5 = 0$

**52.** $3 \sin x = 7 \cos x$

**53.** $5 \sin^2 x - 1 = 0$

**54.** $2 \sin 2x - \cos x = 0$

**55.** $3 \sin^2 x - 7 \sin x + 2 = 0$

**56.** $\tan^4 x - 13 \tan^2 x + 36 = 0$

**57–60** ■ Graph $f$ and $g$ on the same axes, and find their points of intersection.

**57.** $f(x) = 3 \cos x + 1, \quad g(x) = \cos x - 1$

**58.** $f(x) = \sin 2x, \quad g(x) = 2 \sin 2x + 1$

**59.** $f(x) = \tan x, \quad g(x) = \sqrt{3}$

**60.** $f(x) = \sin x - 1, \quad g(x) = \cos x$

**61–64** ■ Use an addition or subtraction formula to simplify the equation. Then find all solutions in the interval $[0, 2\pi)$.

**61.** $\cos x \cos 3x - \sin x \sin 3x = 0$

**62.** $\cos x \cos 2x + \sin x \sin 2x = \frac{1}{2}$

**63.** $\sin 2x \cos x + \cos 2x \sin x = \sqrt{3}/2$

**64.** $\sin 3x \cos x - \cos 3x \sin x = 0$

**65–68** ■ Use a double- or half-angle formula to solve the equation in the interval $[0, 2\pi)$.

**65.** $\sin 2x + \cos x = 0$

**66.** $\tan \dfrac{x}{2} - \sin x = 0$

**67.** $\cos 2x + \cos x = 2$

**68.** $\tan x + \cot x = 4 \sin 2x$

**69–72** ■ Solve the equation by first using a sum-to-product formula.

**69.** $\sin x + \sin 3x = 0$

**70.** $\cos 5x - \cos 7x = 0$

**71.** $\cos 4x + \cos 2x = \cos x$

**72.** $\sin 5x - \sin 3x = \cos 4x$

**73–78** ■ Use a graphing device to find the solutions of the equation, correct to two decimal places.

**73.** $\sin 2x = x$

**74.** $\cos x = \dfrac{x}{3}$

**75.** $2^{\sin x} = x$

**76.** $\sin x = x^3$

**77.** $\dfrac{\cos x}{1 + x^2} = x^2$

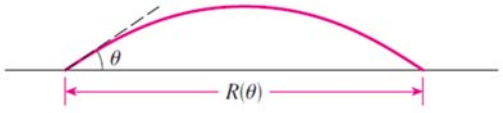
**78.** $\cos x = \frac{1}{2}(e^x + e^{-x})$

## Applications

**79. Range of a Projectile** If a projectile is fired with velocity $v_0$ at an angle $\theta$, then its *range*, the horizontal distance it travels (in feet), is modeled by the function

$$R(\theta) = \frac{v_0^2 \sin 2\theta}{32}$$

(See page 818.) If $v_0 = 2200$ ft/s, what angle (in degrees) should be chosen for the projectile to hit a target on the ground 5000 ft away?




**80. Damped Vibrations** The displacement of a spring vibrating in damped harmonic motion is given by

$$y = 4e^{-3t} \sin 2\pi t$$

Find the times when the spring is at its equilibrium position ($y = 0$).

**81. Refraction of Light** It has been observed since ancient times that light refracts or "bends" as it travels from one medium to another (from air to water, for example). If $v_1$ is