Polynomial:

$P(x) = 2x^3 + 5x^2 - 6x - 9$ ✓

Possible Rational Zeros:

$= \dfrac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

$P(-1) = 0$ so,

$$\begin{array}{r|rrrr} -1 & 2 & 5 & -6 & -9 \\ & & -2 & -3 & 9 \\ \hline & 2 & 3 & -9 & 0 \end{array}$$

$(x+1)(2x^2+3x-9)$

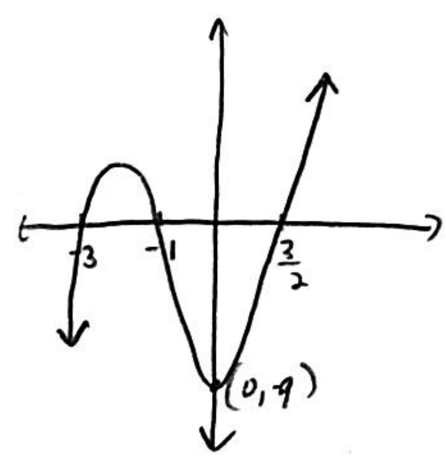
$(x+1)(2x^2+6x-3x-9)$

$(x+1)(2x(x+3)-3(x+3))$

$(x+1)(2x-3)(x+3)$

Complete Factorization:

$(x+1)(2x-3)(x+3)$



Complete Solution Set:

$-1, \frac{3}{2}, -3$

Check:

$- \quad + \quad - \quad +$; $-3 \quad -1 \quad 3/2$

y-int: $(0,-9)$

Polynomial:

$P(x) = 3x^3 - 5x^2 + 2x - 8$

Possible Rational Zeros:

$\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

$P(2) = 0$ so,

$$\begin{array}{r|rrrr} 2 & 3 & -5 & 2 & -8 \\ & & 6 & 2 & 8 \\ \hline & 3 & 1 & 4 & 0 \end{array}$$

$(x-2)(3x^2 + x + 4)$

To solve $3x^2 + x + 4 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(4)}}{2(3)}$

$x = \frac{-1 \pm \sqrt{-47}}{6}$

$x = \frac{-1 \pm i\sqrt{47}}{6}$

Complete Factorization:

$(x-2)(3x^2 + x + 4)$

Complete Solution Set:

$2, \frac{-1 \pm i\sqrt{47}}{6}$

Check:

you would multiply out the factors to show that it would equal $P(x)$

Polynomial:

$P(x) = 2x^5 + 5x^4 - 8x^3 - 14x^2 + 6x + 9$ ✓

Possible Rational Zeros:

$= \dfrac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

$$\begin{array}{r|rrrrrr} -1 & 2 & 5 & -8 & -14 & 6 & 9 \\ & & -2 & -3 & 11 & 3 & -9 \\ \hline -1 & 2 & 3 & -11 & -3 & 9 & 0 \\ & & -2 & -1 & 12 & -9 & \\ \hline & 2 & 1 & -12 & 9 & 0 & \end{array}$$

$f(1) = 0$ so

$$\begin{array}{r|rrrr} 1 & 2 & 1 & -12 & 9 \\ & & 2 & 3 & -9 \\ \hline & 2 & 3 & -9 & 0 \end{array}$$

$(x+1)^2(2x^2+3x-9)$

$(x+1)^2(2x^2+6x-3x-9)$

$(x+1)^2(2x(x+3)-3(x+3))$

$(x+1)^2(2x-3)(x+3)$

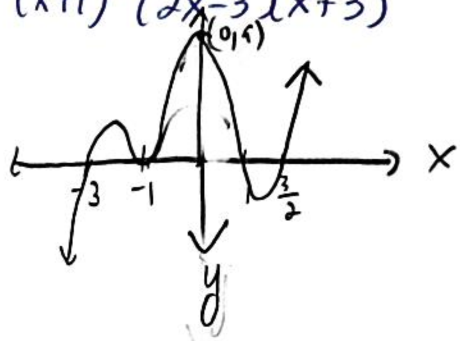
we know that -1 is a double root so

Complete Factorization:

$(x+1)^2(2x-3)(x+3)(x-1)$

Complete Solution Set:

-1 (double root), $3/2$, -3, 1



Check:

$-$	-3	$+$	-1	$+$	1	$-$	$3/2$	$+$

y-int: $(0, 9)$

Polynomial: ✓

$P(x) = 3x^4 - 2x^3 - x^2 - 12x - 4$

Possible Rational Zeros:

$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

They told you that 2 is a zero:

$$\begin{array}{r|rrrrr} 2 & 3 & -2 & -1 & -12 & -4 \\ & & 6 & 8 & 14 & 4 \\ \hline & 3 & 4 & 7 & 2 & 0 \end{array}$$

$P(-\frac{1}{3}) = 0$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 4 & 7 & 2 \\ & & -1 & -1 & -2 \\ \hline & 3 & 3 & 6 & 0 \end{array}$$

Divide by 3

$(x^2 + x + 2)(x-2)(3x+1)$

$(x-2)(3x+1)$

To solve: $x^2 + x + 2 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{-7}}{2}$

$x = \frac{-1 \pm i\sqrt{7}}{2}$

Complete Factorization:

$(3x+1)(x-2)(x^2 + x + 2)$

Complete Solution Set:

$2, -\frac{1}{3}, \frac{-1 \pm i\sqrt{7}}{2}$

Check:

Polynomial:

$$P(x) = x^5 - 9x^4 + 31x^3 - 49x^2 + 36x - 10$$

Possible Rational Zeros:

$$= \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1} = \pm 1, \pm 2, \pm 5, \pm 10$$

Since 1 is a triple root, ...

$$\begin{array}{r|rrrrrr}
1 & 1 & -9 & 31 & -49 & 36 & -10 \\
 & & 1 & -8 & 23 & -26 & 10 \\
\hline
1 & 1 & -8 & 23 & -26 & 10 & 0 \\
 & & 1 & -7 & 16 & -10 & \\
\hline
1 & 1 & -7 & 16 & -10 & 0 & \\
 & & 1 & -6 & 10 & & \\
\hline
 & 1 & -6 & 10 & 0 & &
\end{array}$$

$$(x^2 - 6x + 10)(x - 1)^3$$

To solve $x^2 - 6x + 10 = 0$,

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-4}}{2}$$

$$x = \frac{6 \pm 2i}{2} = 3 \pm i$$

Complete Factorization:

$$(x-1)^3 (x^2 - 6x + 10)$$

Complete Solution Set:

1 (triple root), $3 \pm i$

Check:

Polynomial: ✓

$P(x) = 2x^5 + 7x^4 - 18x^2 - 8x + 8$

Possible Rational Zeros:

$= \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

Given zeros $\frac{1}{2}$ and -2

$$\begin{array}{r|rrrrrr} \frac{1}{2} & 2 & 7 & 0 & -18 & -8 & 8 \\ & & 1 & 4 & 2 & -8 & -8 \\ \hline & 2 & 8 & 4 & -16 & -16 & 0 \end{array}$$

Divide by 2

$$\begin{array}{r|rrrrr} -2 & 1 & 4 & 2 & -8 & -8 \\ & & -2 & -4 & 4 & 8 \\ \hline -2 & 1 & 2 & -2 & -4 & 0 \\ & & -2 & 0 & 4 & \\ \hline & 1 & 0 & -2 & 0 & \end{array}$$

$\rightarrow (x^2-2)(x+2)^2(2x-1)$

To solve $x^2 - 2 = 0$

$x^2 - 2 = 0$

$x^2 = 2$

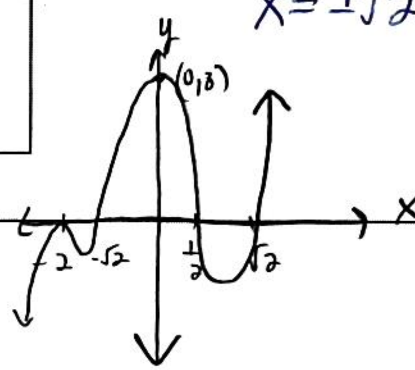
$x = \pm\sqrt{2}$

Complete Factorization:

$(2x-1)(x+2)^2(x^2-2)$

Complete Solution Set:

$\frac{1}{2}$, -2 (double root), $\pm\sqrt{2}$



Check:

$P(x)$: $-$ | -2 | $-$ | $-\sqrt{2}$ | $+$ | $\frac{1}{2}$ | $-$ | $\sqrt{2}$ | $+$

y-int: (0,8)

Polynomial:

$P(x) = 4x^3 + 20x^2 - 23x + 6$ ✓

Possible Rational Zeros:

$\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

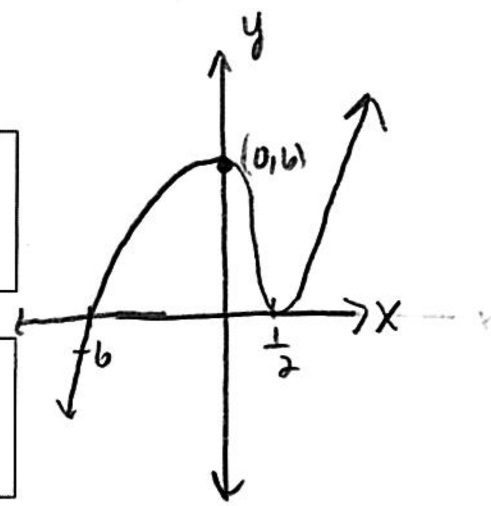
$$\begin{array}{r|rrrr} -6 & 4 & 20 & -23 & 6 \\ & & -24 & 24 & -6 \\ \hline & 4 & -4 & 1 & 0 \end{array}$$

$(4x^2 - 4x + 1)(x+6)$

$(4x^2 - 2x - 2x + 1)(x+6)$

$(2x(2x-1) - 1(2x-1))(x+6)$

$(2x-1)(2x-1)(x+6)$



Complete Factorization:

$(x+6)(2x-1)^2$

Complete Solution Set:

$-6, \frac{1}{2}$ (double root)

Check:

$$\begin{array}{ccccc} - & & + & & + \\ & -6 & & \frac{1}{2} & \end{array}$$

y-int: (0,6)

Polynomial:

$P(x) = 3x^4 - 13x^3 + 7x^2 - 13x + 4$

Possible Rational Zeros:

$= \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \frac{\pm 4}{3}$

$$\begin{array}{r|rrrrr} \frac{1}{3} & 3 & -13 & 7 & -13 & 4 \\ & & 1 & -4 & 1 & -4 \\ \hline & 3 & -12 & 3 & -12 & 0 \end{array}$$

Now divide by 3

$P(4) = 0$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 1 & -4 \\ & & 4 & 0 & 4 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$(3x-1)(x-4)(x^2+1)$

Complete Factorization:

$(3x-1)(x-4)(x^2+1)$

To solve $x^2+1=0$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

Complete Solution Set:

$\frac{1}{3}, 4, \pm i$

Check: