Solve

$$\frac{450}{x} - 4 - 0.10x < 0 \qquad \text{Subtract 54}$$

$$\frac{450 - 4x - 0.10x^2}{x} < 0 \qquad \text{Common denominator}$$

$$\frac{4500 - 40x - x^2}{x} < 0 \qquad \text{Multiply by 10}$$

$$\frac{(90 + x)(50 - x)}{x} < 0 \qquad \text{Factor numerator}$$

		−90		0		50	
Sign of $90 + x$	−		+		+		+
Sign of $50 - x$	+		+		+		−
Sign of x	−		−		+		+
Sign of $\frac{(90 + x)(50 - x)}{x}$	+		−		+		−

The sign diagram shows that the solution of the inequality is $(-90, 0) \cup (50, \infty)$. Because we cannot have a negative number of students, it follows that the group must have more than 50 students for the total cost per person to be less than \$54. ■

1.7 Exercises

1–6 ■ Let $S = \{-2, -1, 0, \frac{1}{2}, 1, \sqrt{2}, 2, 4\}$. Determine which elements of S satisfy the inequality.

1. $3 - 2x \le \frac{1}{2}$

2. $2x - 1 \ge x$

3. $1 < 2x - 4 \le 7$

4. $-2 \le 3 - x < 2$

5. $\frac{1}{x} \le \frac{1}{2}$

6. $x^2 + 2 < 4$

7–28 ■ Solve the linear inequality. Express the solution using interval notation and graph the solution set.

7. $2x - 5 > 3$

8. $3x + 11 < 5$

9. $7 - x \ge 5$

10. $5 - 3x \le -16$

11. $2x + 1 < 0$

12. $0 < 5 - 2x$

13. $3x + 11 \le 6x + 8$

14. $6 - x \ge 2x + 9$

15. $\frac{1}{2}x - \frac{2}{3} > 2$

16. $\frac{2}{5}x + 1 < \frac{1}{5} - 2x$

17. $\frac{1}{3}x + 2 < \frac{1}{6}x - 1$

18. $\frac{2}{3} - \frac{1}{2}x \ge \frac{1}{6} + x$

19. $4 - 3x \le -(1 + 8x)$

20. $2(7x - 3) \le 12x + 16$

21. $2 \le x + 5 < 4$

22. $5 \le 3x - 4 \le 14$

23. $-1 < 2x - 5 < 7$

24. $1 < 3x + 4 \le 16$

25. $-2 < 8 - 2x \le -1$

26. $-3 \le 3x + 7 \le \frac{1}{2}$

27. $\frac{1}{6} < \frac{2x - 13}{12} \le \frac{2}{3}$

28. $-\frac{1}{2} \le \frac{4 - 3x}{5} \le \frac{1}{4}$

29–62 ■ Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

29. $(x + 2)(x - 3) < 0$

30. $(x - 5)(x + 4) \ge 0$

31. $x(2x + 7) \ge 0$

32. $x(2 - 3x) \le 0$

33. $x^2 - 3x - 18 \le 0$

34. $x^2 + 5x + 6 > 0$

35. $2x^2 + x \ge 1$

36. $x^2 < x + 2$

37. $3x^2 - 3x < 2x^2 + 4$

38. $5x^2 + 3x \ge 3x^2 + 2$

39. $x^2 > 3(x + 6)$

40. $x^2 + 2x > 3$

41. $x^2 < 4$

42. $x^2 \ge 9$

43. $-2x^2 \le 4$

44. $(x + 2)(x - 1)(x - 3) \le 0$

45. $x^3 - 4x > 0$

46. $16x \le x^3$

47. $\frac{x - 3}{x + 1} \ge 0$

48. $\frac{2x + 6}{x - 2} < 0$

49. $\frac{4x}{2x + 3} > 2$

50. $-2 < \frac{x + 1}{x - 3}$

51. $\dfrac{2x+1}{x-5} \le 3$

52. $\dfrac{3+x}{3-x} \ge 1$

53. $\dfrac{4}{x} < x$

54. $\dfrac{x}{x+1} > 3x$

55. $1 + \dfrac{2}{x+1} \le \dfrac{2}{x}$

56. $\dfrac{3}{x-1} - \dfrac{4}{x} \ge 1$

57. $\dfrac{6}{x-1} - \dfrac{6}{x} \ge 1$

58. $\dfrac{x}{2} \ge \dfrac{5}{x+1} + 4$

59. $\dfrac{x+2}{x+3} < \dfrac{x-1}{x-2}$

60. $\dfrac{1}{x+1} + \dfrac{1}{x+2} \le 0$

61. $x^4 > x^2$

62. $x^5 > x^2$

63–76 ■ Solve the absolute value inequality. Express the answer using interval notation and graph the solution set.

63. $|x| \le 4$

64. $|3x| < 15$

65. $|2x| > 7$

66. $\frac{1}{2}|x| \ge 1$

67. $|x - 5| \le 3$

68. $|x + 1| \ge 1$

69. $|2x - 3| \le 0.4$

70. $|5x - 2| < 6$

71. $\left|\dfrac{x-2}{3}\right| < 2$

72. $\left|\dfrac{x+1}{2}\right| \ge 4$

73. $|x + 6| < 0.001$

74. $3 - |2x + 4| \le 1$

75. $8 - |2x - 1| \ge 6$

76. $7|x + 2| + 5 > 4$

77–80 ■ A phrase describing a set of real numbers is given. Express the phrase as an inequality involving an absolute value.

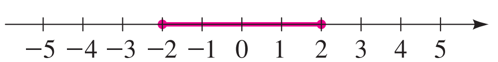
77. All real numbers x less than 3 units from 0

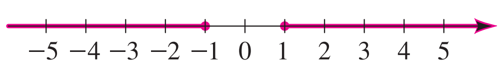
78. All real numbers x more than 2 units from 0

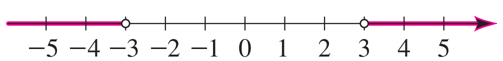
79. All real numbers x at least 5 units from 7

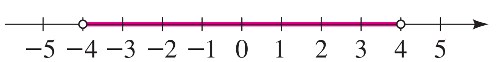
80. All real numbers x at most 4 units from 2

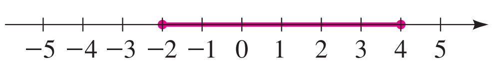
81–86 ■ A set of real numbers is graphed. Find an inequality involving an absolute value that describes the set.

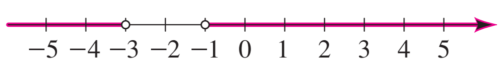
81.


82.


83.


84.


85.


86.


87–90 ■ Determine the values of the variable for which the expression is defined as a real number.

87. $\sqrt{16 - 9x^2}$

88. $\sqrt{3x^2 - 5x + 2}$

89. $\left(\dfrac{1}{x^2 - 5x - 14}\right)^{1/2}$

90. $\sqrt[4]{\dfrac{1-x}{2+x}}$

91. Solve the inequality for x, assuming that a, b, and c are positive constants.

(a) $a(bx - c) \ge bc$ **(b)** $a \le bx + c < 2a$

92. Suppose that a, b, c, and d are positive numbers such that

$$\frac{a}{b} < \frac{c}{d}$$

Show that $\dfrac{a}{b} < \dfrac{a+c}{b+d} < \dfrac{c}{d}$

Applications

93. Temperature Scales Use the relationship between C and F given in Example 9 to find the interval on the Fahrenheit scale corresponding to the temperature range $20 \le C \le 30$.

94. Temperature Scales What interval on the Celsius scale corresponds to the temperature range $50 \le F \le 95$?

95. Car Rental Cost A car rental company offers two plans for renting a car.

Plan A: $30 per day and 10¢ per mile

Plan B: $50 per day with free unlimited mileage

For what range of miles will plan B save you money?

96. Long-Distance Cost A telephone company offers two long-distance plans.

Plan A: $25 per month and 5¢ per minute

Plan B: $5 per month and 12¢ per minute

For how many minutes of long-distance calls would plan B be financially advantageous?

97. Driving Cost It is estimated that the annual cost of driving a certain new car is given by the formula

$$C = 0.35m + 2200$$

where m represents the number of miles driven per year and C is the cost in dollars. Jane has purchased such a car, and decides to budget between $6400 and $7100 for next year's driving costs. What is the corresponding range of miles that she can drive her new car?

98. Gas Mileage The gas mileage g (measured in mi/gal) for a particular vehicle, driven at v mi/h, is given by the formula $g = 10 + 0.9v - 0.01v^2$, as long as v is between 10 mi/h and 75 mi/h. For what range of speeds is the vehicle's mileage 30 mi/gal or better?