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PCH: Rational Zeros and Intermediate Value Theorems

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Do Now:

1. When a function  $f(x)$  is divided by  $2x - 3$ , the quotient is  $3x^2 - 4x + 2$  and remainder is  $-7$ . Find  $f(x)$  in simplest form.

2. Find the remainder when  $x^{124} - 5x^{76} + 2x^{45} - 3x + 5$  is divided by  $x + 1$ .

### **Rational Zeros Theorem**

If the polynomial  $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  has integer coefficients, then every rational zero of  $P$  is of the form

$$\frac{p}{q}$$

where  $p$  is a factor of the constant coefficient  $a_0$   
and  $q$  is a factor of the leading coefficient  $a_n$

**Classwork:**

1. Let  $P(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$ . Find the zeros of  $P(x)$ .

2. Factor the polynomial  $P(x) = 2x^3 + x^2 - 13x + 6$

For 3 - 8, find the complete factorization and all zeros of the following polynomials using the information given.

3.  $P(x) = 2x^5 - 5x^4 + x^3 + 4x^2 - 4x$

4.  $P(x) = x^4 + 6x^3 + 2x^2 - 18x - 15$

5.  $P(x) = x^4 - 5x^3 + 3x^2 + 15x - 18$

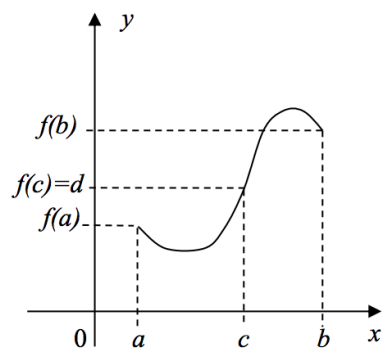
6.  $P(x) = x^4 + 6x^3 + 7x^2 - 12x - 18$

7.  $P(x) = x^4 + 3x^3 + 3x^2 + x$

8.  $P(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

### Intermediate Value Theorem

Let  $a$  and  $b$  be real numbers such that  $a < b$ . If  $f$  is a polynomial function such that  $f(a) \neq f(b)$ , then in the interval  $[a, b]$ ,  $f$  takes on every value between  $f(a)$  and  $f(b)$ .



This theorem helps locate the real zeros of a polynomial function. If  $f(a)$  is positive real number, and another  $f(b)$  is a negative number and  $a < b$ , you can conclude that the function has at least one real zero between these two variables

9. Use the Intermediate Value Theorem to prove that a zero exists on the interval  $[1, 2]$  of the function  $f(x) = -x^3 + 2x^2 + 9x - 11$ .
  
  
  
  
  
  
  
  
  
  
10. Use the Intermediate Value Theorem to prove that  $f(x) = x^3 + x$  takes on the value 9 for some  $x$  in  $[1, 2]$ .

11. Selected value of the continuous function  $f$  are shown in the table below. Is the following statement true or false?

$f(x) = 2$  has at least 1 solution in the interval  $[0, 7]$ .

$x$	$f(x)$
0	4
3	1
4	-4
5	-12
7	-32

12. Selected value of the continuous function  $f$  are shown in the table below. Is the following statement true or false?

$f(x) = 5$  has at least 1 solution in the interval  $[-3, 2]$ .

$x$	$f(x)$
-3	-2
0	10
1	11
2	8