Name:________________________________

AP Calculus AB: Related Rates

Procedures

1. Draw a diagram(s), if possible. In this diagram,
   (a) Label, with letters, any quantities which change at any time, ever.
   (b) Label, with numerical values, only those quantities fixed throughout, never changing. (Substituting other values now makes the problem totally unsolvable.)
2. Specify "Given" and "Fixed."
   (a) Find an equation relating the letters or quantities that appear in the diagram or in the problem. Substitute any "Fixed throughout" values now, and simplify equation to extent possible.
   (b) If "too many" letters appear, it is often useful to use the data of the problem to write an additional equation or equation(s). (For example, the Pythagorean Theorem or the distance formula and/or proportions obtained from similar triangles are often useful.)
3. Differentiate equation(s) implicitly with respect to time t. (Life may be easier if you can rewrite the equation in terms of a single variable.)
4. After differentiating, substitute numbers for quantities whose values are known at a specific time but which are not fixed throughout.
5. Check for hidden values at the specific time (such as values which can be obtained from Pythagorean Theorem, the distance formula, or similar triangles).
6. Interpret the solution. Translate your mathematical result into the problem setting, ***with appropriate units***, and decide whether the result makes sense.

Formulas for Reference

	Volume	Surface Area
Sphere	$V=\frac{4}{3}\pi r^3$	$A=4\pi r^2$
Right Circular Cylinder	$V=\pi r^2 h$	Lateral Surface Area = $2\pi rh$ (For total Area, + top and bottom to above)
Right Circular Cone	$V=\frac{1}{3}\pi r^2 h$	
Cube	$V=e^3$	Total Surface Area = $6e^2$
Rectangular Solid	$V=lwh$	Total Surface Area = $2(lw+lh+wh)$

Adapted from S.Conrad and S.Weiss

Related Rates Problems

1. A stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5m/sec, how fast is the disturbed area growing when r = 8m?

2. The radius of a right circular cone increases at 3 m/sec while the height decreases so that the volume is always 12π m^3. How fast is the height changing when r = 3m?

3. If the volume of an expanding cube is increasing at the rate of $4\ \text{cm}^3/\text{min}$, how fast is its surface area increasing when the surface area is $24\ \text{cm}^2$?

4. Point P moves from the origin along a curve.
   (a) If the equation of the curve is $y = x^3 - 3x^2$, and P's x-coordinate increases at the rate of 3 units/sec., find the rate at which the distance from P to the origin is increasing when P is at (1, -2).
   (b) If the equation of the curve is $y^2 = x^3$, and P's distance from the origin increases at the rate of 2 units/sec., find dx/dt at $(2, 2\sqrt{2})$.

5. A large spherical ball is inflated by a pump which injects helium into the balloon at the rate of $10\ \text{m}^3/\text{sec}$. At the instant when the balloon contains $972\pi\ \text{m}^3$ of gas, how fast is its radius increasing?

6. A 10ft ladder is leaning against a wall. The foot of the ladder is moving away from the wall at 2 ft/sec.
   (a) How fast is the ladders top falling when the ladder is 6 feet from the wall?
   (b) How fast is the top falling when the top is 0.01 feet from the ground ? Does this meet with your intuition?

7. A 6 ft man walks away from a 15ft lamppost at 4 ft/sec..
   (a) How fast is the far end of the man's shadow moving away from the lamppost?
   (b) How fast is the length of his shadow increasing?

8. At noon a ship is sailing due north at the uniform rate of 15 mph. Another ship, 30 miles due north of the first ship, is sailing due east at the constant rate of 20 mph. At what rate is the distance between the ships changing at the end of 1 hour?

9. A tank shaped like an inverted cone is 12m deep and the radius at its brim is 3m. Water is being poured into the tank at $2\,\text{m}^3/\text{min}$.
   (a) How fast is the surface level rising when the water is 4m deep?
   (b) How fast is the surface radius increasing when the volume of the water is $4.5\pi\ \text{m}^3$?

10. The volume of an expanding sphere is increasing at the rate of $10\,\text{m}^3/\text{sec}$. How fast is the surface area of the sphere increasing when the volume is $36\pi\ \text{m}^3$?

11. A point moves along the curve $y^2 = x^3 - x^2$. As it passes through the point (2,2), its x –coordinate is increasing at the rate of 3 units/sec.
   (a) What is the rate of increase of its y-coordinate then?
   (b) How fast is the slope of the graph at the point changing at that instant?

12. The height of a right circular cylinder is increasing at 2 cm/sec. The radius is 6cm. The volume is increasing at $96\pi\ \text{cm}^3/\text{sec}$. How fast is the lateral surface area increasing at that instant? (NOTE: You are told the value of dV/dt, not of V.)

13. A rope connects a dock to the deck of a boat, 3m beneath the level of the dock.

(a) If the rope is being reeled in at 6 m/sec, how fast is the boat approaching the dock when the boat is 4 m from the dock?

(b) If the boat is drifting away from the dock at 2 m/sec, how fast is the rope moving out when the boat is 12m from the dock?