Name:______________________________________ Date:_______

PC: Remainder Theorem and Factor Theorem Ms. Loughran

Do now:

1. Let $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$.
   (a) Find the quotient and remainder when $P(x)$ is divided by $x + 2$.
   (b) Find $P(-2)$.

Remainder Theorem:

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

1. Let $P(x) = x^3 - 2x^2 + 3x - 1$. Find $P(3)$ using 2 different methods.

Factor Theorem:

A polynomial $P(x)$ has a factor of $x - c$ if and only if $P(c) = 0$.

2. Show that $x - 2$ is a factor of $P(x) = x^3 - 3x^2 + 7x - 10$.

3. (a) Use the factor theorem to show that $x+3$ is a factor of $P(x)=x^3-x^2-8x+12$.
(b) Factor $P(x)$ completely.

4. Let $P(x)=x^3-7x+6$.
(a) Show that $P(1)=0$.
(b) Factor $P(x)$ completely.

5. Find a polynomial of degree 4 that has zeros $-3, 0, 1,$ and 5.

Practice Section A

1–6 ■ Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$, and express P in the form $P(x) = D(x) \cdot Q(x) + R(x)$.

1. $P(x) = 3x^2 + 5x - 4, \quad D(x) = x + 3$

2. $P(x) = x^3 + 4x^2 - 6x + 1, \quad D(x) = x - 1$

3. $P(x) = 2x^3 - 3x^2 - 2x, \quad D(x) = 2x - 3$

4. $P(x) = 4x^3 + 7x + 9, \quad D(x) = 2x + 1$

5. $P(x) = x^4 - x^3 + 4x + 2, \quad D(x) = x^2 + 3$

6. $P(x) = 2x^5 + 4x^4 - 4x^3 - x - 3, \quad D(x) = x^2 - 2$

7–12 ■ Two polynomials P and D are given. Use either synthetic or long division to divide $P(x)$ by $D(x)$, and express the quotient $P(x)/D(x)$ in the form

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

7. $P(x) = x^2 + 4x - 8, \quad D(x) = x + 3$

8. $P(x) = x^3 + 6x + 5, \quad D(x) = x - 4$

9. $P(x) = 4x^2 - 3x - 7, \quad D(x) = 2x - 1$

10. $P(x) = 6x^3 + x^2 - 12x + 5, \quad D(x) = 3x - 4$

11. $P(x) = 2x^4 - x^3 + 9x^2, \quad D(x) = x^2 + 4$

12. $P(x) = x^5 + x^4 - 2x^3 + x + 1, \quad D(x) = x^2 + x - 1$

13–22 ■ Find the quotient and remainder using long division.

13. $\dfrac{x^2 - 6x - 8}{x - 4}$

14. $\dfrac{x^3 - x^2 - 2x + 6}{x - 2}$

15. $\dfrac{4x^3 + 2x^2 - 2x - 3}{2x + 1}$

16. $\dfrac{x^3 + 3x^2 + 4x + 3}{3x + 6}$

17. $\dfrac{x^3 + 6x + 3}{x^2 - 2x + 2}$

18. $\dfrac{3x^4 - 5x^3 - 20x - 5}{x^2 + x + 3}$

19. $\dfrac{6x^3 + 2x^2 + 22x}{2x^2 + 5}$

20. $\dfrac{9x^2 - x + 5}{3x^2 - 7x}$

21. $\dfrac{x^6 + x^4 + x^2 + 1}{x^2 + 1}$

22. $\dfrac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8}$

23–36 ■ Find the quotient and remainder using synthetic division.

23. $\dfrac{x^2 - 5x + 4}{x - 3}$

24. $\dfrac{x^2 - 5x + 4}{x - 1}$

25. $\dfrac{3x^2 + 5x}{x - 6}$

26. $\dfrac{4x^2 - 3}{x + 5}$

27. $\dfrac{x^3 + 2x^2 + 2x + 1}{x + 2}$

28. $\dfrac{3x^3 - 12x^2 - 9x + 1}{x - 5}$

29. $\dfrac{x^3 - 8x + 2}{x + 3}$

30. $\dfrac{x^4 - x^3 + x^2 - x + 2}{x - 2}$

31. $\dfrac{x^5 + 3x^3 - 6}{x - 1}$

32. $\dfrac{x^3 - 9x^2 + 27x - 27}{x - 3}$

33. $\dfrac{2x^3 + 3x^2 - 2x + 1}{x - \frac{1}{2}}$

34. $\dfrac{6x^4 + 10x^3 + 5x^2 + x + 1}{x + \frac{2}{3}}$

35. $\dfrac{x^3 - 27}{x - 3}$

36. $\dfrac{x^4 - 16}{x + 2}$

37–49 ■ Use synthetic division and the Remainder Theorem to evaluate $P(c)$.

37. $P(x) = 4x^2 + 12x + 5, \quad c = -1$

38. $P(x) = 2x^2 + 9x + 1, \quad c = \frac{1}{2}$

39. $P(x) = x^3 + 3x^2 - 7x + 6, \quad c = 2$

40. $P(x) = x^3 - x^2 + x + 5, \quad c = -1$

41. $P(x) = x^3 + 2x^2 - 7, \quad c = -2$

42. $P(x) = 2x^3 - 21x^2 + 9x - 200, \quad c = 11$

43. $P(x) = 5x^4 + 30x^3 - 40x^2 + 36x + 14, \quad c = -7$

44. $P(x) = 6x^5 + 10x^3 + x + 1, \quad c = -2$

45. $P(x) = x^7 - 3x^2 - 1, \quad c = 3$

46. $P(x) = -2x^6 + 7x^5 + 40x^4 - 7x^2 + 10x + 112, \quad c = -3$

47. $P(x) = 3x^3 + 4x^2 - 2x + 1, \quad c = \frac{2}{3}$

48. $P(x) = x^3 - x + 1, \quad c = \frac{1}{4}$

49. $P(x) = x^3 + 2x^2 - 3x - 8, \quad c = 0.1$

50. Let

$$P(x) = 6x^7 - 40x^6 + 16x^5 - 200x^4 - 60x^3 - 69x^2 + 13x - 139$$

Calculate $P(7)$ by **(a)** using synthetic division and **(b)** substituting $x = 7$ into the polynomial and evaluating directly.

51–54 ■ Use the Factor Theorem to show that $x - c$ is a factor of $P(x)$ for the given value(s) of c.

51. $P(x) = x^3 - 3x^2 + 3x - 1, \quad c = 1$

52. $P(x) = x^3 + 2x^2 - 3x - 10, \quad c = 2$

53. $P(x) = 2x^3 + 7x^2 + 6x - 5, \quad c = \frac{1}{2}$

54. $P(x) = x^4 + 3x^3 - 16x^2 - 27x + 63, \quad c = 3, -3$

55–56 ■ Show that the given value(s) of c are zeros of $P(x)$, and find all other zeros of $P(x)$.

55. $P(x) = x^3 - x^2 - 11x + 15, \quad c = 3$

56. $P(x) = 3x^4 - x^3 - 21x^2 - 11x + 6, \quad c = \frac{1}{3}, -2$

57–60 ■ Find a polynomial of the specified degree that has the given zeros.

57. Degree 3; zeros $-1, 1, 3$

58. Degree 4; zeros $-2, 0, 2, 4$

59. Degree 4; zeros $-1, 1, 3, 5$

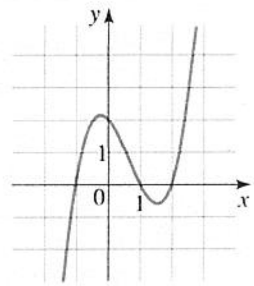
60. Degree 5; zeros $-2, -1, 0, 1, 2$

61. Find a polynomial of degree 3 that has zeros 1, -2, and 3, and in which the coefficient of x^2 is 3.

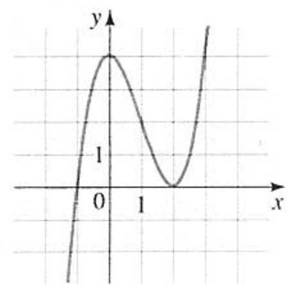
62. Find a polynomial of degree 4 that has integer coefficients and zeros 1, -1, 2, and $\frac{1}{2}$.

63–66 ■ Find the polynomial of the specified degree whose graph is shown.

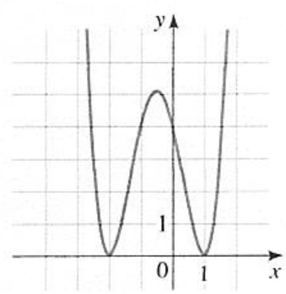
63. Degree 3



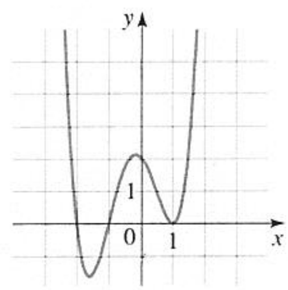
64. Degree 3



65. Degree 4



66. Degree 4



Discovery • Discussion

67. Impossible Division? Suppose you were asked to solve the following two problems on a test:

A. Find the remainder when $6x^{1000} - 17x^{562} + 12x + 26$ is divided by $x + 1$.

B. Is $x - 1$ a factor of $x^{567} - 3x^{400} + x^9 + 2$?

Practice Section B

Use synthetic division and the remainder theorem.

1. $f(x) = x^3 - x^2 + 3x - 2$; find $f(2)$.
2. $f(x) = 2x^3 + 3x^2 - x - 5$; find $f(-1)$.
3. $f(x) = x^4 - 3x^2 + x + 2$; find $f(3)$.
4. $f(x) = x^4 + 2x^3 - 3x - 1$; find $f(-2)$.
5. $f(x) = x^5 - x^3 + 2x^2 + x - 3$; find $f(1)$.
6. $f(x) = 3x^4 + 2x^3 - 3x^2 - x + 7$; find $f(-3)$.

Find the remainder for each division by substitution, using the remainder theorem. That is, in Exercise 7 (for example) let $f(x) = x^3 - 2x^2 + 3x - 5$ and find $f(2) = r$.

7. $(x^3 - 2x^2 + 3x - 5) \div (x - 2)$
8. $(x^3 - 2x^2 + 3x - 5) \div (x + 2)$
9. $(2x^3 + 3x^2 - 5x + 1) \div (x - 3)$
10. $(3x^4 - x^3 + 2x^2 - x + 1) \div (x + 3)$
11. $(4x^5 - x^3 - 3x^2 + 2) \div (x + 1)$
12. $(3x^5 - 2x^4 + x^3 - 7x + 1) \div (x - 1)$

Show that the given binomial $x - c$ is a factor of $p(x)$, and then factor $p(x)$ completely.

13. $p(x) = x^3 + 6x^2 + 11x + 6$; $x + 1$
14. $p(x) = x^3 - 6x^2 + 11x - 6$; $x - 1$
15. $p(x) = x^3 + 5x^2 - 2x - 24$; $x - 2$
16. $p(x) = -x^3 + 11x^2 - 23x - 35$; $x - 7$
17. $p(x) = -x^3 + 7x + 6$; $x + 2$
18. $p(x) = x^3 + 2x^2 - 13x + 10$; $x + 5$
19. $p(x) = 6x^3 - 25x^2 - 29x + 20$; $x - 5$
20. $p(x) = 12x^3 - 22x^2 - 100x - 16$; $x + 2$
21. $p(x) = x^4 + 4x^3 + 3x^2 - 4x - 4$; $x + 2$
22. $p(x) = x^4 - 8x^3 + 7x^2 + 72x - 144$; $x - 4$
23. $p(x) = x^6 + 6x^5 + 8x^4 - 6x^3 - 9x^2$; $x + 3$