Polynomial:
$P(x) = x^6 - x^5 - 8x^4 + 14x^3 + x^2 - 13x + 6$

Possible Rational Zeros:

$\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 6$

$P(-3) = 0$
$P(-1) = 0$
$P(1) = 0$
$P(2) = 0$

-3	1	-1	-8	14	1	-13	6
		-3	12	-12	-6	15	-6
-1	1	-4	4	2	-5	2	0
		-1	5	-9	7	-2	
1	1	-5	9	-7	2	0	
		1	-4	5	-2		
2	1	-4	5	-2	0		
		2	-4	2			
	1	-2	1	0			

$x^2 - 2x + 1$
$(x-1)^2$

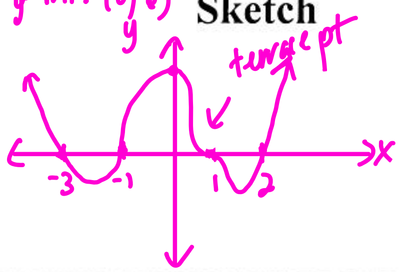
Complete Factorization:
$(x+3)(x+1)(x-1)^3(x-2)$

Complete Solution Set:
$\{-3, -1, 2, 1 \text{ (triple)}\}$

$P(x)$: + | -3 | − | -1 | + | 1 | − | 2 | +

y-int: (0, 6)

Sketch



Key to Review Sheet for Exam 2
Q2

$P(2)=0$

or

① $2\,|\;1\quad -3\quad -10\quad 24$
$\quad\;\; 2\quad -2\quad -24$
$\quad\;\; 1\quad -1\quad -12\quad 0$

therefore $x-2$ is a factor

$(x-2)(x^2-x-12)$
$(x-2)\boxed{(x-4)(x+3)}$
other factors

② $P(x)=3x^{107}+14x^{35}-16x$
$P(1)=3(1)^{107}+14(1)^{35}-16(1)$
$=3+14-16$
$=1$

③ $P(x)=14x^{10}-2x^3-17$
$P(-2)=14(-2)^{10}-2(-2)^3-17$
$=14335$

④ $f(-3)=(-3)^3+(-3)^2-5(-3)+3$
$f(-3)=0$
so $x+3$ is a factor of $f(x)$

⑤ $f(-1)=(-1)^3-13(-1)^2+23(-1)-11$
$=-1-13-23-11\neq 0$
so $x+1$ is not a factor of $f(x)$

⑦ $-16=x$

⑧ $3/2=x$

⑨ $x=3, \frac{1}{2}, -3$

⑩ $x-8$

⑪ $2x-3$

⑥ (a) $\dfrac{\pm 1}{\pm 1, \pm 3} = \pm 1, \pm \frac{1}{3}$

(b) $\dfrac{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64}{\pm 1} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$

(c) $\dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm 2/3, \pm 4/3, \pm 8/3$

(d) $\dfrac{\pm 1, \pm 5, \pm 25}{\pm 1, \pm 2, \pm 4, \pm 8} = \pm 1, \pm 5, \pm 25, \pm\frac{1}{2}, \pm 5/2, \pm\frac{25}{2}, \pm\frac{1}{4}, \pm\frac{5}{4}, \pm\frac{25}{4}, \pm\frac{1}{8}, \pm\frac{5}{8}, \pm\frac{25}{8}$

(12) (a)

$$\begin{array}{r|rrrrr} 7 & 5 & -46 & 84 & -50 & 7 \\ & & 35 & -77 & 49 & -7 \\ \hline 1 & 5 & -11 & 7 & -1 & \cancel{0} \\ & & 5 & -6 & +1 & \\ \hline & 5 & -6 & 1 & 0 & \end{array}$$

$\left\{ \frac{1}{5}, 7, 1 \text{ (double zero)} \right\}$

$5x^2 - 6x + 1 = 0$

$5x^2 - 5x - x + 1 = 0$

$5x(x-1) - 1(x-1) = 0$

$(5x-1)(x-1) = 0$

$x = 1/5 \quad x = 1$

(b)

$$\begin{array}{r|rrrr} -3/2 & 2 & 9 & 19 & 15 \\ & & -3 & -9 & -15 \\ \hline & \underbrace{2 \quad 6 \quad 10}_{\div 2} & & & 0 \end{array}$$

$\left\{ -3/2, \frac{-3 \pm i\sqrt{11}}{2} \right\}$

$(x^2 + 3x + 5)(2x + 3) = 0$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2(1)} = \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm i\sqrt{11}}{2}$

(13) (a)

$$\begin{array}{r|rrrr} -1 & 3 & 11 & 5 & -3 \\ & & -3 & -8 & 3 \\ \hline & 3 & 8 & -3 & 0 \end{array}$$

$(x+1)(3x^2 + 8x - 3)$

$(x+1)(3x^2 + 9x - x - 3)$

$(x+1)(3x(x+3) - 1(x+3))$

$(x+1)(3x-1)(x+3)$

(13) (b)

-1	3	-10	-24	-6	5
		-3	13	11	-5
-1	3	-13	-11	5	$\cancel{0}$
		-3	16	-5	
	3	-16	5	0	

$$(x+1)^2(3x^2-16x+5)$$
$$(x+1)^2(3x^2-15x-x+5)$$
$$(x+1)^2(3x(x-5)-1(x-5))$$
$$(x+1)^2(3x-1)(x-5)$$

(14)/(15) a) $f(x)=x^3+3x^2-10x-24$

possible rational zeros: $\dfrac{\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24}{\pm1} = \pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

$f(1)=1^3+3(1)^2-10(1)-24\neq0$

$f(-1)=(-1)^3+3(-1)^2-10(-1)-24\neq0$

$f(2)=2^3+3(2)^2-10(2)-24 = 8+24-20-24\neq0$

$f(-2)=(-2)^3+3(-2)^2-10(-2)-24 = -8+24+20-24\neq0$

$f(3)=3^3+3(3)^2-10(3)-24 = 27+27-30-24=0$ $\therefore$ 3 is a zero and $(x-3)$ is a factor

3	1	3	-10	-24
		3	18	24
	1	6	8	0

x^2+6x+8

$(x+4)(x+2)$

Complete factorization: $(x-3)(x+4)(x+2)$

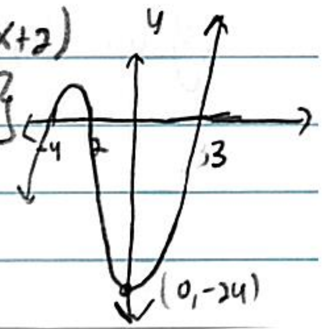
Complete solution set: $\{3,-4,-2\}$

sketch:

$-\quad+\quad-\quad+$ f

$-4\quad -2\quad 3$

y int: $(0,-24)$



b) $f(x) = 2x^3 + 3x^2 - 23x - 12$

possible rational zeros: $\dfrac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1, \pm2} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{3}{2}$

$f(1) = 2(1)^3 + 3(1)^2 - 23(1) - 12 = 2 + 3 - 23 - 12 \neq 0$

$f(-1) = 2(-1)^3 + 3(-1)^2 - 23(-1) - 12 = -2 + 3 + 23 - 12 \neq 0$

$f(2) = 2(2)^3 + 3(2)^2 - 23(2) - 12 = 16 + 12 - 46 - 12 \neq 0$

$f(-2) = 2(-2)^3 + 3(-2)^2 - 23(-2) - 12 \neq 0$

$f(3) = 2(3)^3 + 3(3)^2 - 23(3) - 12 = 54 + 27 - 69 - 12 = 0 \therefore$ 3 is a zero and $(x-3)$ is a factor

$$\begin{array}{r|rrrr} 3 & 2 & 3 & -23 & -12 \\ & & 6 & 27 & 12 \\ \hline & 2 & 9 & 4 & 0 \end{array}$$

$2x^2 + 9x + 4$

$2x^2 + 8x + x + 4$

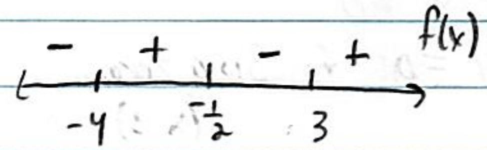
$2x(x+4) + 1(x+4)$

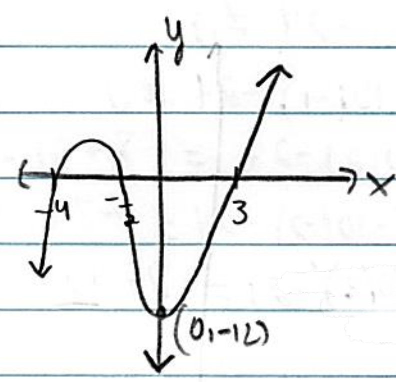
$(2x+1)(x+4)$

complete factorization: $(x-3)(2x+1)(x+4)$

complete solution set: $\{3, -\frac{1}{2}, -4\}$

sketch:





y-int: $(0, -12)$

ⓒ $f(x) = x^4 + 3x^3 - x^2 - 7x - 4$

possible rational zeros: $\frac{\pm 1, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4$

$f(1) = 1^4 + 3(1)^3 - 1^2 - 7(1) - 4 \neq 0$

$f(-1) = (-1)^4 + 3(-1)^3 - (-1)^2 - 7(-1) - 4 = 1 - 3 - 1 + 7 - 4 = 0$

$\therefore$ -1 is a zero
(x+1) is a factor

$$\begin{array}{r|rrrrr}
-1 & 1 & 3 & -1 & -7 & -4 \\
 & & -1 & -2 & 3 & 4 \\
\hline
-1 & 1 & 2 & -3 & -4 & 0 \\
 & & -1 & -1 & 4 & \\
\hline
 & 1 & 1 & -4 & 0 &
\end{array}$$

$f(2) \neq 0$
$f(-2) \neq 0$
$f(4) \neq 0$
$f(-4) \neq 0$

since none of the other possible rational zeros are zeros of this polynomial I am going to see if -1 is a double root.

$x^2 + x - 4$
not factorable

complete factorization: $(x+1)^2(x^2+x-4)$

Complete solution set: $\left\{-1 \text{ (double)}, \frac{-1 \pm \sqrt{17}}{2}\right\}$

$$x = \frac{-1 \pm \sqrt{(+1)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{-17}}{2}$$

(d) $f(x) = x^5 - 6x^4 + 11x^3 - 2x^2 - 12x + 8$

possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1} = \pm 1, \pm 2, \pm 4, \pm 8$

$f(1) = 1 - 6 + 11 - 2 - 12 + 8 = 0$
$f(-1) = -1 - 6 - 11 - 2 + 12 + 8 = 0$
$f(2) = 32 - 96 + 88 - 8 - 24 + 8 = 0$

$\therefore \pm 1$ and 2 are zeros and
$(x+1)(x-1)(x-2)$ are factors

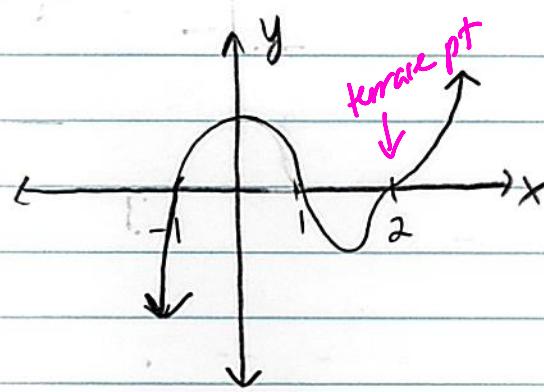
$$\begin{array}{r|rrrrrr}
1 & 1 & -6 & 11 & -2 & -12 & 8 \\
 & & 1 & -5 & 6 & 4 & -8 \\
\hline
-1 & 1 & -5 & 6 & 4 & -8 & 0 \\
 & & -1 & 6 & -12 & 8 & \\
\hline
2 & 1 & -6 & 12 & -8 & 0 & \\
 & & 2 & -8 & 8 & & \\
\hline
 & 1 & -4 & 4 & 0 & &
\end{array}$$

complete factorization: $(x-2)^3(x+1)(x-1)$
complete solution set: $\{\pm 1, 2 \text{ (triple)}\}$

$x^2 - 4x + 4$
$(x-2)(x-2)$

$$\begin{array}{ccccccc}
- & & + & & - & & + \quad f(x) \\
\longleftarrow & | & & | & & | & \longrightarrow \\
 & -1 & & 1 & & 2 &
\end{array}$$

y-int: $(0, 8)$



(e) $f(x) = x^5 - 11x^3 + 28x = x(x^4 - 11x^2 + 28)$

GCF
AM tweaking

$f(x) = x(x^2 - 7)(x^2 - 4)$

$f(x) = x(x-2)(x+2)(x^2-7)$

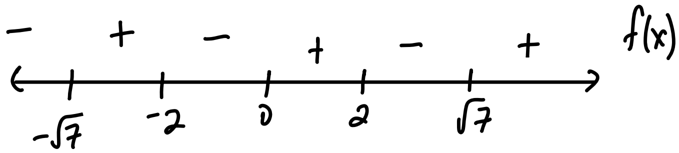
complete factorization: $f(x) = x(x-2)(x+2)(x^2-7)$

complete solution set: $\{0, \pm 2, \pm\sqrt{7}\}$

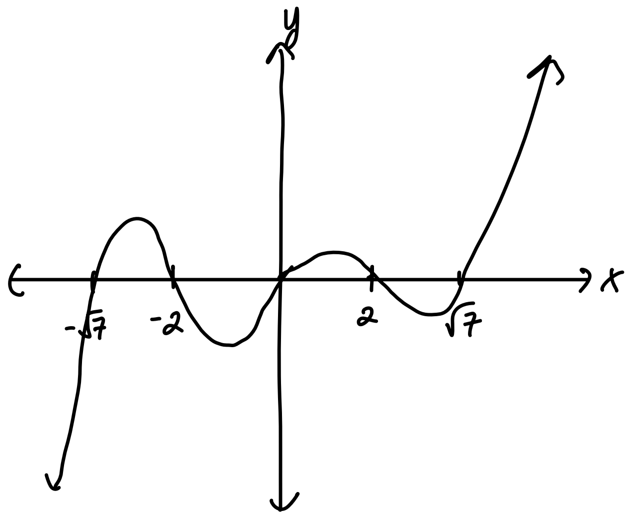
$x^2 - 7 = 0$

$x^2 = 7$

$x = \pm\sqrt{7}$



y-int: (0,0)



(quotient)(divisor) + remainder

(16) $(3x^2 - 2x + 2)(2x + 3) + 5$

$6x^3 - 4x^2 + 4x + 9x^2 - 6x + 6 + 5$

$6x^3 + 5x^2 - 2x + 11$

(17) $(2x^2 + x - 3)(3x - 1) - 4$

$6x^3 + 3x^2 - 9x - 2x^2 - x + 3 - 4$

$6x^3 + x^2 - 10x - 1$