The General Equation $ax^2 + by^2 = c$

Depending on the values of the coefficients, the general equation $ax^2 + by^2 = c$, where $a, b, c \neq 0$, describes the graph of a *circle*, *ellipse*, or *hyperbola*.

Values of Coefficients	Name of Graph	Example	
$a = b$ and have the same sign as c	circle	$2x^2 + 2y^2 = 18$ or $x^2 + y^2 = 9$ circle with center at origin and radius = 3	
$a \neq b$ and have the same sign as c	ellipse	$9x^2 + 25y^2 = 225$ ellipse with center at origin and x-intercepts = ± 5 y-intercepts = ± 3	
a, b have different signs	hyperbola	$x^2 - y^2 = 9$ hyperbola with center at origin and x-intercepts = ± 3 no y-intercepts	

Recall: The equation of a parabola contains only one square term: either $y = ax^2 + bx + c$ or $x = ay^2 + by + c$
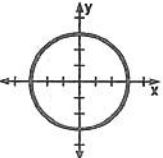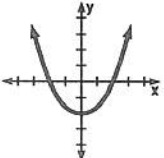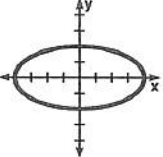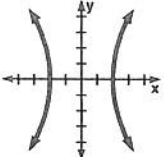The equation of a straight line contains no square terms: $ax + by = c$

EXERCISES

In 1-14, identify the graph of the given relation as
(1) a circle
(2) an ellipse
(3) a hyperbola
(4) a parabola

1. $4y^2 = 25 - 4x^2$
2. $2x^2 + 3y^2 = 24$
3. $x^2 = y^2 + 9$
4. $x^2 = 6 - y$
5. $4x^2 - 100 = 25y^2$
6. $3y^2 = 6 - x^2$
7. $3x^2 + 2y^2 = 6$
8. $4x^2 + 16y^2 = 25$
9. $x^2 + y = 9$
10. $2x^2 = 5 - 2y^2$
11. $y^2 = 6 - 3x^2$
12. $2x^2 - 9 = 2y^2$
13. $4x^2 - 4y^2 = 9$
14. $x^2 - \frac{y^2}{16} = 1$

15. Which of the following is the equation of a hyperbola?
(1) $x^2 = 10 - y^2$
(2) $x = y^2 - 9$
(3) $y^2 = x^2 - 1$
(4) $4x^2 + y^2 = 9$

16. The graph of which equation is an ellipse?
(1) $3x^2 - 4y^2 = 7$
(2) $\frac{y + 6}{x - 1} = 3$
(3) $y = 2x^2 + 3x - 5$
(4) $x^2 + 5y^2 = 2$

17. Which is an equation of a circle?
(1) $2x^2 - 2y^2 = 18$
(2) $2x^2 + 3y^2 = 36$
(3) $3x^2 + 3y^2 = 21$
(4) $x^2 = y^2 + 16$

18. Which equation has a hyperbola as its graph?
(1) $x^2 = 10 + y$
(2) $x^2 = 10 - y^2$
(3) $3x^2 = 10 - 2y^2$
(4) $3x^2 = 10 + 2y^2$

19. Which equation has an ellipse as its graph?
(1) $2x^2 = 8 - 3y$
(2) $2x^2 = 8 + 3y^2$
(3) $2x^2 = 8 - 3y^2$
(4) $2x = 8 - 3y$

20. Which is an equation of a circle?
(1) $2x^2 + y^2 = 7$
(2) $x = \frac{y}{8}$
(3) $x^2 - y^2 = 10$
(4) $5(x^2 + y^2) = 12$

21. Which is an equation of a parabola?
 (1) $x^2 = 3 + y^2$ (3) $x = 3 + y$
 (2) $x = 3 + y^2$ (4) $y^2 = 3x^2 + 3$

22. The graph of the relation $ay = bx^2 + c$ in which neither a nor b is 0 is
 (1) a parabola (3) an ellipse
 (2) a straight line (4) a hyperbola

23. If a, b, and c are positive unequal numbers, the graph of $ax^2 + by^2 = c$ is
 (1) a circle (3) an ellipse
 (2) a parabola (4) a hyperbola

24. The graph of $ax^2 + by^2 = c$, in which a, b, and c are real numbers, is an ellipse if
 (1) $a = b, a > 0, b < 0, c > 0$
 (2) $a = b, a > 0, b > 0, c < 0$
 (3) $a \neq b, a > 0, b > 0, c > 0$
 (4) $a \neq b, a > 0, b < 0, c > 0$

25. If $a \neq 0$, $b \neq 0$, and $c \neq 0$, the graph of $ax^2 + by^2 = c$ can not be
 (1) an ellipse (3) a parabola
 (2) a circle (4) a hyperbola

26. The graph of the equation $\frac{x^2}{4} + \frac{y^2}{16} = 1$ passes through the point whose coordinates are
 (1) $(0, 0)$ (2) $(0, 2)$ (3) $(0, 4)$ (4) $(4, 0)$

27. Which relation is a function?
 (1) $\{(x, y)| x^2 + y = 4\}$ (3) $\{(x, y)| x^2 - y^2 = 4\}$
 (2) $\{(x, y)| x^2 + y^2 = 4\}$ (4) $\{(x, y)| x^2 + 4y^2 = 4\}$

28. If the replacement set is the set of real numbers, what is the domain of the relation represented by $\{(x, y)| x^2 + 4y^2 = 16\}$?
 (1) $\{y| -2 \leq y \leq 2\}$ (3) $\{x| -4 \leq x \leq 4\}$
 (2) $\{y| -2 < y < 2\}$ (4) $\{x| -4 < x < 4\}$

29. Which is the graph of a quadratic relation for which the domain consists of all the real numbers?

30. If the graphs of the equations $x^2 + y^2 = 9$ and $y = 3$ are drawn on the same set of axes, what is the total number of points common to both graphs?
 (1) 1 (2) 2 (3) 3 (4) 0

31. When drawn on the same set of axes, the points of intersection of the graphs of $x^2 + y^2 = 16$ and $x = 2$ are located in quadrants
 (1) I and III (3) II and III
 (2) I and IV (4) II and IV

32. The graphs of the equations $x^2 + y^2 = 25$ and $y = x^2$ are drawn on the same set of axes. The total number of points common to these graphs is
 (1) 1 (2) 2 (3) 3 (4) 4

33. The graph of $x^2 + y^2 = 25$ and the graph of $x - 4 = 0$ are drawn on the same set of axes. A point of intersection of the graphs is
 (1) $(5, 0)$ (2) $(-4, -3)$ (3) $(4, -3)$ (4) $(-3, 4)$

34. What is the graph of the solution set of $x^2 + y^2 > 9$?
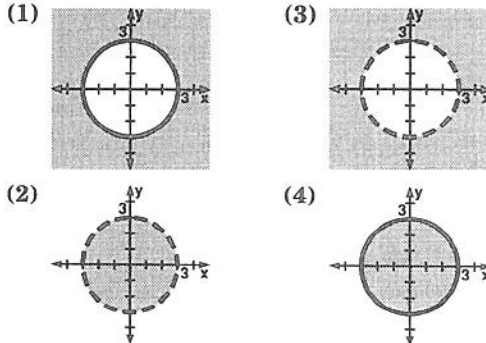
35. Each equation in column A has one of the geometric figures in column B as its graph. List the numbers 1-5 on your answer paper and after each number write the letter that indicates the corresponding graph.

Column A
(1) $x^2 + y^2 - 4 = 0$
(2) $4x^2 + y^2 - 1 = 0$
(3) $x^2 - y - 4 = 0$
(4) $x^2 + 4y^2 = 0$
(5) $x^2 - 4y^2 = 0$

Column B
a. The point $(0, 0)$
b. Two straight lines parallel to the y-axis
c. Two straight lines intersecting at the origin
d. A parabola that crosses the y-axis at $(0, -4)$
e. A circle whose center is the origin and whose radius is 2
f. An ellipse that crosses the y-axis at $(0, 1)$ and $(0, -1)$
g. A hyperbola that crosses the y-axis at $(0, 2)$ and $(0, -2)$