Name:________________________________ Date:___________
PC: Sum and Difference of Angles Formulas Ms. Loughran

Sums and Differences of Angles

Formulas for Sums of Angles	Formulas for Differences of Angles
$\sin(x+y) = \sin x \cos y + \cos x \sin y$	$\sin(x-y) = \sin x \cos y - \cos x \sin y$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$	$\cos(x-y) = \cos x \cos y + \sin x \sin y$
$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$	$\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

One application of these formulas is to prove other identities.

Examples:

1. Find the exact value of tan 15°.

2. Prove that the following is an identity: $\tan x + \tan y = \dfrac{\sin(x+y)}{\cos x \cos y}$

3. Given: $\angle w$ in Quadrant I, $\angle t$ in Quadrant II, $\sin w = \dfrac{2}{3}, \cos t = -\dfrac{4}{5}$. Find $\sin(w+t)$.

Exercises

1. Which equation is not a trigonometric identity?
   (1) $\sin^2 x + \cos^2 x = 1$
   (2) $\tan x = \frac{\sin x}{\cos x}$
   (3) $\cos(x + y) = \cos x \cos y + \sin x \sin y$
   (4) $\sin(x + y) = \sin x \cos y + \cos x \sin y$

2. $\sin(\theta + 270°)$ is equivalent to
   (1) $\cos\theta$ (2) $2\cos\theta$ (3) $-\cos\theta$ (4) $-\sin\theta$

3. $\sin(180° + A)$ is equivalent to
   (1) $\cos A$ (2) $\sin A$ (3) $-\cos A$ (4) $-\sin A$

4. $\sin(90° - \theta)$ is equivalent to
   (1) $\cos\theta$ (2) $\sin\theta$ (3) $-\cos\theta$ (4) $-\sin\theta$

5. $\cos(\theta + 90°)$ is equivalent to
   (1) $\sin\theta$ (2) $\cos\theta$ (3) $-\sin\theta$ (4) $-\cos\theta$

6. $\cos(2\pi - x)$ is equivalent to
   (1) $-\cos x$ (2) $\cos x$ (3) $-\sin x$ (4) $\sin x$

7. $\tan(x + 45°)$ is equivalent to
   (1) $\frac{\tan x - 1}{1 + \tan x}$ (3) $\frac{\tan x}{1 + \tan x}$
   (2) $\frac{\tan x + 1}{1 - \tan x}$ (4) $\frac{\tan x}{1 - \tan x}$

8. $\tan(180° - y)$ is equivalent to
   (1) -1 (3) $-\tan y$
   (2) $\frac{-\tan y}{1 + \tan y}$ (4) $\frac{1 - \tan y}{1 + \tan y}$

9. $\cos(A - B) - \cos(A + B)$ is equivalent to
   (1) $-2\sin A \sin B$ (3) $2\cos A \cos B$
   (2) $-2\cos B$ (4) $2\sin A \sin B$

10. $\frac{\sin(x + y)}{\cos x \cos y}$ is equivalent to
    (1) $1 + \cot x$ (3) $\tan x + \tan y$
    (2) $\tan x + 1$ (4) $\frac{1}{\cos y} + \frac{1}{\cos x}$

In 11–12, use a sum or difference formula to prove that the given statement is an identity.

11. $\sin(-\theta) = -\sin\theta$

12. $\tan(-\theta) = -\tan\theta$

In 13–22, prove that the given statement is an identity for all values of the angles for which the expressions are defined.

13. $\sin(x + 45°) = \frac{\sqrt{2}}{2}(\sin x + \cos x)$

14. $\cos(60° + y) = \frac{1}{2}(\cos y - \sqrt{3}\sin y)$

15. $\tan(45° + x) = \frac{1 + \tan x}{1 - \tan x}$

16. $\tan(45° - B) = \frac{\cos B - \sin B}{\cos B + \sin B}$

17. $\cos(60° + B) + \cos(60° - B) = \frac{1}{\sec B}$

18. $\frac{\sin(A - B)}{\sin A \sin B} = \cot B - \cot A$

19. $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

20. $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$

21. $\frac{\cos(x - y)}{\cos(x + y)} = \frac{\cot x + \tan y}{\cot x - \tan y}$

22. $\frac{\sin(A + B)\cos C}{\sin(A + C)\cos B} = \frac{1 + \cot A \tan B}{1 + \cot A \tan C}$

23. a. Using the formula for $\cos(x - y)$, find the exact value of $\cos 15°$ in radical form if $m\angle x = 45°$ and $m\angle y = 30°$.

    b. Using the formula for $\sin(x - y)$, find the exact value of $\sin 15°$ in radical form if $m\angle x = 45°$ and $m\angle y = 30°$.

    c. Find the exact value of $\sin 75°$, using the formula for $\sin(x - y)$ where $m\angle x = 90°$ and $m\angle y = 15°$. Use the values for $\cos 15°$ and $\sin 15°$ found in parts a and b.

24. Since $\cos 75° = \cos(30° + 45°)$, then $\cos 75°$ equals
    (1) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (3) $\frac{-\sqrt{2} - \sqrt{6}}{4}$
    (2) $\frac{-\sqrt{6} + \sqrt{2}}{4}$ (4) $\frac{\sqrt{2} + \sqrt{6}}{4}$

25. $\sin 35° \cos 22° + \cos 35° \sin 22°$ equals
    (1) $\sin 13°$ (2) $\sin 57°$ (3) $\cos 13°$ (4) $\cos 57°$

26. $\sin 60° \cos 45° - \sin 45° \cos 60°$ equals
    (1) 1 (2) 0 (3) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (4) $\frac{1}{2}$

27. $\cos 70° \cos 40° - \sin 70° \sin 40°$ equals
    (1) $\cos 30°$ (2) $\cos 70°$ (3) $\cos 110°$ (4) $\sin 70°$

28. $\sin 13° \cos 17° + \cos 13° \sin 17°$ equals
    (1) 1 (2) $\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) 0

29. $\sin 42° \cos 48° + \cos 42° \sin 48°$ equals
    (1) 1 (2) 0 (3) $\sin 6°$ (4) $\cos 6°$

30. $\sin 96° \cos 24° + \cos 96° \sin 24°$ equals
    (1) $\sin 60°$ (2) $-\sin 60°$ (3) $\cos 60°$ (4) $-\cos 60°$

31. $\sin 210° \cos 30° - \cos 210° \sin 30°$ equals
    (1) 1 (2) -1 (3) 0 (4) 180

32. Express in radical form:
    $\sin 90° \cos 30° - \cos 90° \sin 30°$

33. If $\sin x = \frac{3}{5}$ and x is a positive acute angle, find $\cos\left(x + \frac{\pi}{2}\right)$.

34. If A and B are positive acute angles and if $\sin A = \frac{3}{5}$ and $\sin B = \frac{4}{5}$, then $\sin (A + B)$ is equal to

(1) 1 (2) 0 (3) $\frac{7}{5}$ (4) $\frac{24}{25}$

35. If x and y are positive acute angles, and $\sin x = \frac{3}{5}$ and $\sin y = \frac{1}{2}$, then $\cos (x + y)$ is equal to

(1) $\frac{4\sqrt{3}+3}{10}$ (3) $\frac{4}{5}+\frac{\sqrt{3}}{2}$

(2) $\frac{4\sqrt{3}-3}{10}$ (4) $\frac{4}{5}-\frac{\sqrt{3}}{2}$

36. If $\tan x = \frac{1}{2}$ and $\tan y = 1$, the value of $\tan (x + y)$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) 3 (4) $\frac{3}{2}$

37. If x and y are positive acute angles, and $\sin x = \frac{3}{5}$ and $\sin y = \frac{1}{2}$, then $\sin (x + y)$ is equal to

(1) $\frac{3\sqrt{3}-4}{10}$ (3) $\frac{12}{25}+\frac{\sqrt{3}}{4}$

(2) $\frac{3\sqrt{3}+4}{10}$ (4) $\frac{12}{25}-\frac{\sqrt{3}}{4}$

38. If $\sin \alpha = \frac{3}{5}$, $\tan \beta = \frac{5}{12}$, and α and β are in the first quadrant, then the value of $\cos (\alpha + \beta)$ is

(1) $-\frac{16}{65}$ (2) $\frac{33}{65}$ (3) $\frac{56}{65}$ (4) $\frac{63}{65}$

39. If $\sin A = \frac{3}{5}$, $\sin B = \frac{5}{13}$, and angles A and B are acute angles, what is the value of $\cos (A - B)$?

(1) $-\frac{12}{65}$ (2) $\frac{16}{65}$ (3) $\frac{33}{65}$ (4) $\frac{63}{65}$

40. If $\tan x = \frac{1}{2}$ and $\tan y = \frac{1}{3}$, then the value of $\tan (x + y)$ is

(1) 1 (2) $\frac{5}{7}$ (3) $\frac{1}{5}$ (4) $\frac{1}{7}$

In 41–44, express the answer in simplest form.

41. If $\tan x = 1$ and $\tan y = 2$, find the value of $\tan (x + y)$.

42. If x and y are obtuse angles such that $\sin x = \frac{3}{5}$ and $\sin y = \frac{1}{2}$, find the value of $\sin (x + y)$.

43. If x and y are positive acute angles such that $\cos x = \frac{12}{13}$ and $\cos y = \frac{4}{5}$, find the value of $\cos (x + y)$.

44. If A and B are positive acute angles such that $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, find the value of $\cos (A + B)$.