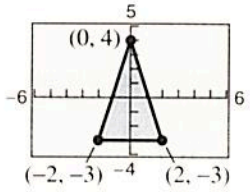
In Exercise 18, Cramer's Rule does not apply.
In Exercises 22, 23, and 26, the points are not collinear.

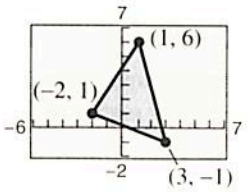
8.5 /// EXERCISES

In Exercises 1–10, use a determinant to find the area of the triangle with the given vertices.

1.



2.



3. (0, 0), (1, 5), (3, 1)

4. (0, 0), (4, 5), (5, −2)

5. $(0, \frac{1}{2})$, $(\frac{5}{2}, 0)$, (4, 3)

6. (0, 4), (2, 3), (5, 0)

7. (4, 5), (6, 1), (7, 9)

8. (0, −2), (−1, 4), (3, 5)

9. (−3, 5), (2, 6), (3, −5)

10. (−2, 4), (1, 5), (3, −2)

In Exercises 11 and 12, find a value of x such that the triangle has an area of 4.

11. (−5, 1), (0, 2), (−2, x)

12. (−4, 2), (−3, 5), (−1, x)

In Exercises 13–16, use Cramer's Rule to solve (if possible) the system of equations.

13.
$$\begin{aligned} 3x + 4y &= -2 \\ 5x + 3y &= 4 \end{aligned}$$

14.
$$\begin{aligned} -0.4x + 0.8y &= 1.6 \\ 0.2x + 0.3y &= 2.2 \end{aligned}$$

15.
$$\begin{aligned} 4x - y + z &= -5 \\ 2x + 2y + 3z &= 10 \\ 5x - 2y + 6z &= 1 \end{aligned}$$

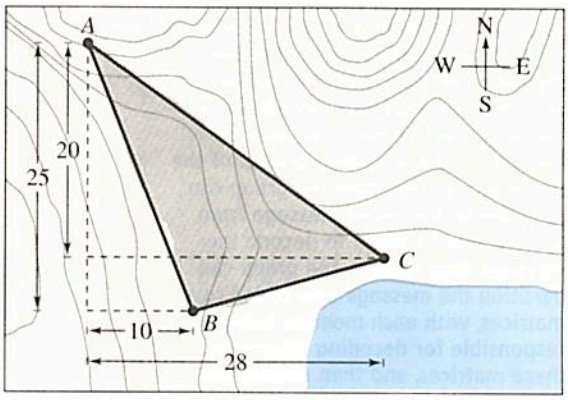
16.
$$\begin{aligned} 4x - 2y + 3z &= -2 \\ 2x + 2y + 5z &= 16 \\ 8x - 5y - 2z &= 4 \end{aligned}$$

In Exercises 17 and 18, use a graphing utility and Cramer's Rule to solve (if possible) the system of equations.

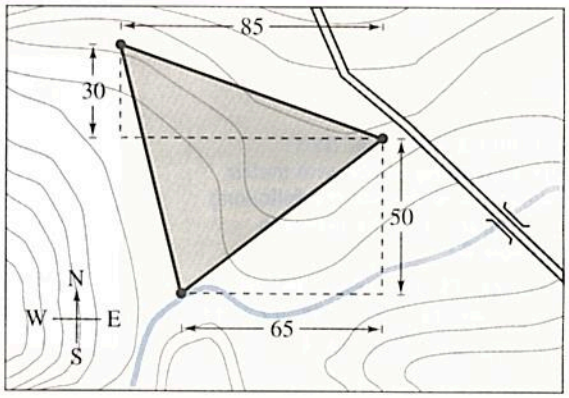
17.
$$\begin{aligned} 3x + 3y + 5z &= 1 \\ 3x + 5y + 9z &= 2 \\ 5x + 9y + 17z &= 4 \end{aligned}$$

18.
$$\begin{aligned} 2x + 3y + 5z &= 4 \\ 3x + 5y + 9z &= 7 \\ 5x + 9y + 17z &= 13 \end{aligned}$$

19. *Area of a Region* A large region of forest has been infected with gypsy moths. The region is roughly triangular, as shown in the figure. From the northernmost vertex A of the region, the distances to the other vertices are 25 miles south and 10 miles east (for vertex B), and 20 miles south and 28 miles east (for vertex C). Use a graphing utility to approximate the number of square miles in this region.



20. *Area of a Region* You own a triangular tract of land, as shown in the figure. To estimate the number of square feet in the tract, you start at one vertex, walk 65 feet east and 50 feet north to the second vertex, and then walk 85 feet west and 30 feet north to the third vertex. Use a graphing utility to determine how many square feet there are in the tract of land.



In Exercises 21–26, use the determinant feature of a graphing utility to decide if the points are collinear.

21. $(3, -1), (0, -3), (12, 5)$

22. $(-3, -5), (6, 1), (10, 2)$

23. $\left(2, -\frac{1}{2}\right), (-4, 4), (6, -3)$

24. $(0, 1), (4, -2), (-8, 7)$

25. $(0, 2), (1, 2.4), (-1, 1.6)$

26. $(2, 3), (3, 3.5), (-1, 2)$

In Exercises 27–32, use a determinant to find an equation of the line through the points.

27. $(0, 0), (5, 3)$

28. $(0, 0), (-2, 2)$

29. $(-4, 3), (2, 1)$

30. $(10, 7), (-2, -7)$

31. $\left(-\frac{1}{2}, 3\right), \left(\frac{5}{2}, 1\right)$

32. $\left(\frac{2}{3}, 4\right), (6, 12)$

In Exercises 33 and 34, find x such that the points are collinear.

33. $(2, -5), (4, x), (5, -2)$

34. $(-6, 2), (-5, x), (-3, 5)$

In Exercises 35 and 36, find the uncoded 1×3 row matrices for the message. Then encode the message using the matrix.

Message	*Matrix*
35. TROUBLE IN RIVER CITY	$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$
36. PLEASE SEND MONEY	$\begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$

In Exercises 37–40, write a cryptogram for the message using the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}.$$

37. LANDING SUCCESSFUL

38. BEAM ME UP SCOTTY

39. HAPPY BIRTHDAY

40. OPERATION OVERLORD

In Exercises 41 and 42, use A^{-1} to decode the cryptogram.

41. $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

11, 21, 64, 112, 25, 50, 29, 53, 23, 46, 40, 75, 55, 92

42. $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$

9, −1, −9, 38, −19, −19, 28, −9, −19, −80, 25, 41, −64, 21, 31, 9, −5, −4

In Exercises 43 and 44, decode the cryptogram by using the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}.$$

43. 20, 17, −15, −12, −56, −104, 1, −25, −65, 62, 143, 181

44. 13, −9, −59, 61, 112, 106, −17, −73, −131, 11, 24, 29, 65, 144, 172

45. The following cryptogram was encoded with a 2×2 matrix.

8, 21, −15, −10, −13, −13, 5, 10, 5, 25,
5, 19, −1, 6, 20, 40, −18, −18, 1, 16

The last word of the message is _RON. What is the message?

46. The following cryptogram was encoded with a 2×2 matrix.

5, 2, 25, 11, −2, −7, −15, −15, 32,
14, −8, −13, 38, 19, −19, −19, 37, 16

The last word of the message is _SUE. What is the message?