Name:__________________________ Date:___________

PC: Trigonometric Identities Ms. Loughran

The Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

You are familiar with the following reciprocal identities:

$$\sec\theta = \frac{1}{\cos\theta}, \cos\theta \neq 0 \qquad \csc\theta = \frac{1}{\sin\theta}, \sin\theta \neq 0 \qquad \cot\theta = \frac{1}{\tan\theta}, \tan\theta \neq 0$$

And the quotient identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \cos\theta \neq 0 \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}, \sin\theta \neq 0$$

An identity is an equation that is true for all permissible replacements of the variable.

Proving an identity:
To prove that a trigonometric statement is an identity, note:

1. **The object is to show that the two sides of the statement are equivalent.**
   - ⇒ You may work on only one side and show that it is equivalent to the other.
     - ➢ Work on the more complicated side.
   - ⇒ You may work on the two sides independently until you arrive at equivalent expressions.
     - ➢ You may not perform operations involving the two sides simultaneously. You are not solving an equation. That is, never cross the equal sign for any purpose. As a reminder, use a line between sides.

2. **Use the basic identities to transform one or both sides of the proposed identity.**
   - ⇒ A general starting point is to rewrite expressions in terms of sine and cosine, but be alert to situations when a Pythagorean substitution is appropriate.

3. **After replacements have been made, do the algebra suggested by the form of the expression.**
   - ⇒ If there is a complex fraction, simplify it.
   - ⇒ If there are two fractions, combine them.
   - ⇒ Look for possibilities of factoring.

Exercise Set A

In 1–29, for all values of the angle for which the expressions are defined, choose an equivalent expression.

1. $\frac{-1}{\cos A}$ is equivalent to
(1) $\sec A$ (2) $-\sec A$ (3) $\sin A$ (4) $-\sin A$

2. $\frac{\cot\theta}{\csc\theta}$ is equivalent to
(1) $\sec\theta$ (2) $\sin\theta$ (3) $\cos\theta$ (4) $\csc\theta$

3. $\frac{\sec\theta}{\csc\theta}$ is equivalent to
(1) $\sin\theta$ (2) $\cos\theta$ (3) $\tan\theta$ (4) $\cot\theta$

4. $\frac{\sin\theta}{\tan\theta}$ is equivalent to
(1) $-\cos\theta$ (3) $1-\cos\theta$
(2) $\cos\theta$ (4) $1+\cos\theta$

5. $\frac{\sin^2 A}{\tan A}$ is equivalent to
(1) $\frac{\sin A}{\cos A}$ (3) $\frac{1}{\sin A\cos A}$
(2) $\sin A\cos A$ (4) $\frac{\cos A}{\sin A}$

6. $\sin\theta$ is equivalent to
(1) $\frac{\tan\theta}{\sec\theta}$ (2) $\frac{1}{\sec\theta}$ (3) $\sec\theta$ (4) $\frac{\sec\theta}{\tan\theta}$

7. The expression $\frac{\tan x}{\sec^2 x}$ is equivalent to
(1) $\sin x$ (3) $\frac{\sin^3 x}{\cos x}$
(2) $\sin x\cos x$ (4) $\frac{\cos^3 x}{\sin x}$

8. $\sqrt{\frac{2\cos^2\theta}{\sin^2\theta}}$ is equivalent to
(1) $2\tan\theta$ (3) $2\cot\theta$
(2) $\sqrt{2}\tan\theta$ (4) $\sqrt{2}\cot\theta$

9. $(\tan\theta)(\csc\theta)$ is equivalent to
(1) $\sin\theta$ (2) $\cos\theta$ (3) $\csc\theta$ (4) $\sec\theta$

10. $(\cot\theta)(\sec\theta)$ is equivalent to
(1) $\tan\theta$ (2) $\cos\theta$ (3) $\cot\theta$ (4) $\csc\theta$

11. $\tan A\cdot\cos A\cdot\csc A$ is equivalent to
(1) 1 (2) $\frac{1}{2}$ (3) $\sin A$ (4) $\frac{1}{\sin A}$

12. $\csc y+1$ is equivalent to
(1) $\frac{\cot y}{\csc y-1}$ (3) $\cot y$
(2) $\frac{\sin y+1}{\sin y}$ (4) $\frac{1+\cos y}{\cos y}$

13. $\sec x-\tan x$ is equivalent to
(1) 1 (3) $\frac{1-\sin x}{\cos x}$
(2) $\cos x-\cot x$ (4) $\frac{\cos x-\sin^2 x}{\sin x\cos x}$

14. $\sin\theta(\csc\theta-\sin\theta)$ is equivalent to
(1) 1 (3) $\tan\theta-1$
(2) $\cos\theta$ (4) $\cos^2\theta$

15. $\cos y(\csc y-\sec y)$ is equivalent to
(1) $\cot y-1$ (3) $1-\tan y$
(2) $\tan y-1$ (4) $-\cos y$

16. $\cot^2\theta$ is equivalent to
(1) $\frac{1}{\sin^2\theta}$ (3) $1-\cos^2\theta$
(2) $\cos^2\theta$ (4) $\frac{\cos^2\theta}{1-\cos^2\theta}$

17. $\frac{\sin^2 x+\cos^2 x}{\cos x}$ is equivalent to
(1) $\sin x\cos x$ (3) $\csc x$
(2) $\tan x\cos x$ (4) $\sec x$

18. $\cos A+\frac{\sin^2 A}{\cos A}$ is equivalent to
(1) 1 (2) $\sec A$ (3) $\csc A$ (4) $\cos A$

19. $4+\cos^2 A$ is equivalent to
(1) $5-\sec^2 A$ (3) $\frac{5}{\sec^2 A}$
(2) $5-\sin^2 A$ (4) $5+\sin^2 A$

20. $\frac{1}{\sin^2 A}-1$ is equivalent to
(1) $\cot^2 A$ (3) $\sec^2 A-1$
(2) $\cos^2 A$ (4) $\frac{\sin^2 A-1}{\sin^2 A}$

21. $\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$ is equivalent to
(1) 1 (2) $\sec\theta$ (3) $\frac{1}{\csc\theta}$ (4) $\frac{1}{\sin\theta\cos\theta}$

22. $\frac{\cot^2 x}{1-\sin^2 x}$ is equivalent to
(1) $\cos^2 x$ (3) $\frac{1}{\sin^2 x}$
(2) $\tan^2 x$ (4) $1-\sin^2 x$

23. $\dfrac{\cos x-\frac{\sin^2 x}{\cos x}}{1+\frac{\sin x}{\cos x}}$ is equivalent to
(1) $\cos x+\sin x$ (3) $\frac{1}{\cos x+\sin x}$
(2) $\cos x-\sin x$ (4) $\frac{1}{\cos x-\sin x}$

24. $\sin\theta\left(\frac{1}{\sin\theta}-\sin\theta\right)$ is equivalent to
(1) $-\cos^2\theta$ (3) $1-\cos^2\theta$
(2) $\cos^2\theta$ (4) $1+\cos^2\theta$

25. $\frac{2(1+\cos A)}{\sin^2 A+\cos A+\cos^2 A}$ is equivalent to

(1) 1 (2) 2 (3) $\frac{2}{\sin A}$ (4) $\frac{2}{\cos A}$

26. $\frac{\cos^2 B}{\sin B}+\sin B$ is equivalent to

(1) 1 (2) $\frac{1}{\csc B}$ (3) $\frac{1}{\sin B}$ (4) $\cos^2 B$

27. $\sin^4 B-\cos^4 B$ is equivalent to

(1) $1+\cos^2 B$ (3) $\sin^2 B+\cos^2 B$

(2) $1-\cos^2 B$ (4) $\sin^2 B-\cos^2 B$

28. $\sec^2 x+\csc^2 x$ is equivalent to

(1) $\sin^2 x\cos^2 x$ (3) $1+\tan^2 x$

(2) $\frac{1}{\sin^2 x\cos^2 x}$ (4) $1-\tan^2 x$

29. $\frac{1-\tan^2\theta}{1+\tan^2\theta+2\tan\theta}$ is equivalent to

(1) $\frac{\tan\theta-1}{\tan\theta+1}$ (3) $\frac{1}{\tan\theta}+1$

(2) $\frac{1-\tan\theta}{1+\tan\theta}$ (4) $1-\frac{1}{\tan\theta}$

30. The expression $\tan x$ is not equivalent to

(1) $\sin x\sec x$ (3) $\cot x\sin x$

(2) $\frac{\sin x}{\cos x}$ (4) $\frac{\cos x\sec x}{\cot x}$

In 31–35, rewrite the expression in terms of $\sin\theta$ and $\cos\theta$. Express the result in simplest form.

31. $\sin\theta\sec\theta\cot\theta$

32. $\frac{\cot\theta}{\csc\theta}$

33. $\frac{1}{\sec^2\theta}+\frac{1}{\csc^2\theta}$

34. $\cot\theta+\tan\theta$

35. $\sec\theta-\tan\theta\sin\theta$

Exercise Set B

In 1–27, prove that the given statement is an identity for all values of the angle for which the expressions are defined.

1. $\sec\theta-\sin\theta\tan\theta=\cos\theta$
2. $\tan\theta+\cot\theta=\sec\theta\csc\theta$
3. $(\sin A+1)(\csc A-1)=\cos A\cot A$
4. $(1+\csc\theta)(1-\sin\theta)=\cot\theta\cos\theta$
5. $\frac{\tan A+\sin A}{\csc A+\cot A}=\sin A\tan A$
6. $\sin^2 x(1+\tan^2 x)=\tan^2 x$
7. $\frac{1}{\tan x-\cot x}=\frac{\sin x\cos x}{2\sin^2 x-1}$
8. $\frac{\cos\theta+\cot\theta}{\cos\theta\cot\theta}=\tan\theta+\sec\theta$
9. $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}=2\cot x\sec x$
10. $1+\frac{1}{\cos x}=\frac{\tan^2 x}{\sec x-1}$
11. $\frac{1+\tan^2\theta}{1-\cos^2\theta}=\sec^2\theta\csc^2\theta$
12. $2\cos^2 x-1=\frac{1-\tan^2 x}{1+\tan^2 x}$
13. $\frac{\cos x}{\tan x}=\csc x(1-\sin^2 x)$
14. $\frac{\cos\theta\sin\theta+\cos\theta}{\cos^2\theta}=\tan\theta+\sec\theta$
15. $\frac{\cos\theta\sin^2\theta}{1-\cos\theta}=\cos\theta+\cos^2\theta$
16. $\frac{\tan\theta-\cot\theta}{\tan\theta+\cot\theta}=2\sin^2\theta-1$
17. $\csc x-\sin x=\frac{\cot x}{\sec x}$
18. $\frac{\tan x\csc^2 x}{1+\tan^2 x}=\cot x$
19. $\frac{\sin x+\tan x}{1+\sec x}=\sin x$
20. $\frac{\sin\theta\tan\theta+\cos\theta}{\cos\theta}=\sec^2\theta$
21. $\frac{\sin\theta\cot\theta+\cos^2\theta}{1+\cos\theta}=\cos\theta$
22. $\frac{\cos\theta}{\sin\theta\tan\theta+\cos\theta}=\frac{1}{\sec^2\theta}$
23. $2\csc^2\theta=\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}$
24. $\cos\theta(\cos\theta+1)+\sin^2\theta=\frac{\sin\theta+\tan\theta}{\tan\theta}$
25. $\frac{\sin x-\cos y}{\sin x+\cos y}=\frac{\sec y-\csc x}{\sec y+\csc x}$
26. $\frac{1-\cos\theta}{\sin\theta}=\frac{\sin\theta}{1+\cos\theta}$
27. $(\tan\theta+\sec\theta)^2=\frac{1+\sin\theta}{1-\sin\theta}$