Do Now:

1.  Graph $y = \dfrac{x^4 - 2x^2 + 1}{x^2 - 1}$. State the domain, range coordinates of any hole(s), $x$- and $y$-intercepts and the equations of any asymptotes.

Let $r$ be the **REDUCED** rational function

$$r(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_1 x + b_0}$$

1.  The vertical asymptotes of $r$ are the lines $x = a$, where $a$ is a zero of the denominator.

    **In other words:**

2.  (a) If $n < m$, then $r$ has a horizontal asymptote of $y = 0$.

    **In other words:**

    (b) If $n = m$, then $r$ has a horizontal asymptote of $y = \dfrac{a_n}{b_m}$.

    **In other words:**

    (c) If $n > m$, then $r$ has no horizontal asymptote.

    **In other words:**

| Function | Hole(s) | Vertical Asymptote(s) | Horizontal Asymptote | $x$-intercept(s) | $y$-intercept |
|---|---|---|---|---|---|
| $y = \dfrac{1-x}{x+3}$ | | | | | |
| $y = \dfrac{x-2}{x^2-4}$ | | | | | |
| $y = \dfrac{x^2-x-20}{x+4}$ | | | | | |
| $y = \dfrac{x^2-x-20}{x+1}$ | | | | | |
| $y = \dfrac{2x^3}{x^3+x}$ | | | | | |
| $y = \dfrac{x-1}{x^2-4}$ | | | | | |

# More Asymptotes and Holes

| Function | Hole(s) | Vertical Asymptote(s) | Horizontal Asymptote | x-intercept(s) | y-intercept |
|---|---|---|---|---|---|
| $y = \dfrac{4x}{x-3}$ | | | | | |
| $y = \dfrac{5x^2}{3+x}$ | | | | | |
| $y = \dfrac{-4x^2}{(x-2)(x+4)}$ | | | | | |
| $y = \dfrac{-4+x^2}{(x-2)(x+4)}$ | | | | | |
| $y = \dfrac{3x(x-1)}{2x^2-5x+3}$ | | | | | |
| $y = \dfrac{x}{x^4-1}$ | | | | | |